Name: ____________________

Algebra 2

Fall 2024 Student Workbook - Pyret Edition

Workbook v0.9-beta

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Introduction to Computational Data Science

Many important questions ("What's the best restaurant in town?", "Is this law good for citizens?", etc.) are answered with *data*.
Data Scientists try to answer these questions by writing *programs that ask questions about data*.

Data of all types can be organized into **Tables**.

- Every Table has a **header row** and some number of **data rows**.
- **Quantitative data** is numeric and measures *an amount*, such as a person's height, a score on a test, distance, etc. A list of quantitative data can be ordered from smallest to largest.
- **Categorical data** is data that specifies *qualities*, such as sex, eye color, country of origin, etc. Categorical data is not subject to the laws of arithmetic — for example, we cannot take the "average" of a list of colors.

Categorical or Quantitative?

- **Quantitative data** measures an *amount* and can be ordered from smallest to largest.
- **Categorical data** specifies *qualities* and is not subject to the laws of arithmetic — for example, we cannot take the "average" of a list of colors.

  Note: Numbers can sometimes be categorical rather than quantitative!

For each piece of data below, circle whether it is **Categorical** or **Quantitative**.

1)	Hair color	categorical	quantitative
2)	Age	categorical	quantitative
3)	ZIP Code	categorical	quantitative
4)	Date	categorical	quantitative
5)	Height	categorical	quantitative
6)	Sex	categorical	quantitative
7)	Street Name	categorical	quantitative

For each question, circle whether it will be answered by **Categorical** or **Quantitative** data.

8)	We'd like to find out the average price of cars in a lot.	categorical	quantitative
9)	We'd like to find out the most popular color for cars.	categorical	quantitative
10)	We'd like to find out which puppy is the youngest.	categorical	quantitative
11)	We'd like to find out which cats have been fixed.	categorical	quantitative
12)	We want to know which people have a ZIP code of 02907.	categorical	quantitative

★ We decide to sort the animals in *ascending order* (smallest-to-largest) by `age`. Then we sort the table in *alphabetical order* (A-to-Z) by `name`.

Does that mean `name` is a quantitative column? Why or why not? ______________________________

____

____

____

Questions and Column Descriptions

1) Take some time to look through the Animals Dataset. What stands out to you? Which animals are interesting? What patterns do you notice? Put your observations in the **Notice** column below.

2) Do any of these observations make you wonder? If so, write your question next to the observation in the **Wonder** column. If not, think of another question to write down.

Notice	Wonder	Answered by this dataset?
I notice that *Kujo took a long time to be adopted*	*Is it because he was so big?*	*Yes No*
I notice that		*Yes No*
I notice that		*Yes No*
I notice that		*Yes No*
I notice that		*Yes No*
I notice that		*Yes No*
I notice that		*Yes No*

Describe the table, and two of the columns, by filling in the blanks below.

1. This dataset is about ______________________________; it contains ________ data rows.
2. Some of the columns are:
   a. ______________ (column name), which contains ______________ (*categorical or quantitative*) data. Some example values are:
   ______________________________.
   b. ______________ (column name), which contains ______________ (*categorical or quantitative*) data. Some example values are:
   ______________________________.

Introduction to Programming

The **Editor** is a software program we use to write Code. Our Editor allows us to experiment with Code on the right-hand side, in the **Interactions Area**. For Code that we want to *keep*, we can put it on the left-hand side in the **Definitions Area**. Clicking the "Run" button causes the computer to re-read everything in the Definitions Area and erase anything that was typed into the Interactions Area.

Data Types

Programming languages involve different ***data types***, such as Numbers, Strings, Booleans, and even Images.

- Numbers are values like `1`, `0.4`, `1/3`, and `-8261.003`.
  - Numbers are *usually* used for quantitative data and other values are *usually* used as categorical data.
  - In Pyret, any decimal *must* start with a 0. For example, `0.22` is valid, but `.22` is not.
- Strings are values like `"Emma"`, `"Rosanna"`, `"Jen and Ed"`, or even `"08/28/1980"`.
  - All strings *must* be surrounded by quotation marks.
- Booleans are either `true` or `false`.

All values evaluate to themselves. The program `42` will evaluate to `42`, the String `"Hello"` will evaluate to `"Hello"`, and the Boolean `false` will evaluate to `false`.

Operators

Operators (like `+`, `-`, `*`, `<`, etc.) work the same way in Pyret that they do in math.

- Operators are written between values, for example: `4 + 2`.
- In Pyret, operators must always have spaces around them. `4 + 2` is valid, but `4+2` is not.
- If an expression has different operators, parentheses must be used to show order of operations. `4 + 2 + 6` and `4 + (2 * 6)` are valid, but `4 + 2 * 6` is not.

Applying Functions

Applying functions works much the way it does in math. Every function has a name, takes some inputs, and produces some output. The function name is written first, followed by a list of ***arguments*** in parentheses.

- In math this could look like $f(5)$ or $g(10, 4)$.
- In Pyret, these examples would be written as `f(5)` and `g(10, 4)`.
- Applying a function to make images would look like `star(50, "solid", "red")`.
- There are many other functions, for example `num-sqr`, `num-sqrt`, `triangle`, `square`, `string-repeat`, etc.

Functions have ***contracts***, which help explain how a function should be used. Every Contract has three parts:

- The *Name* of the function - literally, what it's called.
- The *Domain* of the function - what *type(s) of value(s)* the function consumes, and in what order.
- The *Range* of the function - what *type of value* the function produces.

Strings and Numbers

*Make sure you've loaded code.pyret.org (CPO), clicked "Run", and are working in the **Interactions Area** on the right. Hit Enter/return to evaluate expressions you test out.*

Strings

String values are always in quotes.

- Try typing your name *(in quotes!)*.
- Try typing a sentence like "I'm excited to learn to code!" *(in quotes!)*.
- Try typing your name with the opening quote, but *without the closing quote.* Read the error message!
- Now try typing your name *without any quotes.* Read the error message!

1) Explain what you understand about how strings work in this programming language. ______________________

____

Numbers

2) Try typing `42` into the Interactions Area and hitting "Enter". Is `42` the same as `"42"` ? Why or why not?

____

3) What is the largest number the editor can handle?

____

4) Try typing `0.5` . Then try typing `.5` . Then try clicking on the answer. Experiment with other decimals.

Explain what you understand about how decimals work in this programming language. ______________________

____

5) What happens if you try a fraction like `1/3` ? ______________________

____

6) Try writing **negative** integers, fractions and decimals. What do you learn? ______________________

____

____

Operators

7) Just like math, Pyret has ***operators*** like `+` , `-` , `*` and `/` .
Try typing in `4 + 2` and then `4+2` (without the spaces). What can you conclude from this?

____

8) Type in the following expressions, **one at a time**: `4 + 2 * 6` `(4 + 2) * 6` `4 + (2 * 6)` What do you notice?

____

9) Try typing in `4 + "cat"` , and then `"dog" + "cat"` . What can you conclude from this?

____

____

Booleans

Boolean-producing expressions are yes-or-no questions, and will always evaluate to either `true` ("yes") or `false` ("no").

What will the expressions below evaluate to? *Write down your prediction, then type the code into the Interactions Area to see what it returns.*

	Prediction	Result		Prediction	Result
1) `3 <= 4`			2) `"a" > "b"`		
3) `3 == 2`			4) `"a" < "b"`		
5) `2 < 4`			6) `"a" == "b"`		
7) `5 >= 5`			8) `"a" <> "a"`		
9) `4 >= 6`			10) `"a" >= "a"`		
11) `3 <> 3`			12) `"a" <> "b"`		
13) `4 <> 3`			14) `"a" >= "b"`		

15) In your own words, describe what `<` does. ______________________________

16) In your own words, describe what `>=` does. ______________________________

17) In your own words, describe what `<>` does. ______________________________

	Prediction:	Result:
18) `string-contains("catnap", "cat")`		
19) `string-contains("cat", "catnap")`		

20) In your own words, describe what `string-contains` does. Can you generate another expression using `string-contains` that returns true?

____

★ There are infinite string values ("a", "aa", "aaa"...) and infinite number values out there (...-2,-1,0,-1,2...). But how many different *Boolean* values are there? ______________________________

Functions for Tables

Open the [Animals Starter File] and click "Run".
In the Interactions Window on the right, type `animals-table` and hit "Enter" to see the default view of the table.

sort

Suppose we wanted to see the names of the animals in alphabetical order…

The `sort` function takes in three pieces of information:

1. A table
2. A column we want to sort the table by (declared using a String)
3. The order in which we want the column sorted (declared using a Boolean)

Test out these two expressions in the Interactions Area and record what you learn about ordering below:

- `sort(animals-table, "species", true)`
- `sort(animals-table, "species", false)`

1) `true` sorts the table… ____________________

2) `false` sorts the table… ____________________

Suppose we wanted to sort the `animals-table` by the `weeks` column to determine which animals were adopted quickest…

3) Would you use `true` or `false`? Explain. ____________________

4) Test it out, and write your thinking about ***quantitative*** columns at the end of your explanations of `true` and `false` above.

5) Which animal(s) were adopted the quickest? ____________________

6) Some functions produce Numbers, some produce Strings, some produce Booleans. What did the `sort` function produce? __________

There are many other functions available to us in Pyret. We can describe them using contracts. The Contract for `sort` is:

```
# sort :: Table, String, Boolean -> Table
```

- Each Contract begins with the function name: *in this case* `sort`
- Lists the data types required to satisfy its Domain: *in this case* Table, String, Boolean
- And then declares the data type of the Range it will return. *in this case* Table
- Contracts can also be written with more detail, by adding *variable names* in the Domain:

```
# sort :: (Table, String, Boolean) -> Table
```
table-name, column-name, order

Suppose we wanted to sort the `animals-table` by the `legs` column to determine which animals had the most legs…

7) Fill in the blanks below with the code you'd use (We've put pieces of the Contract below each line to help you!):

__________ (__________ , __________ , __________)
function-name table-name :: Table column-name :: String order :: Boolean

8) Which animal(s) had the most legs? ____________________

9) Think of another question you might answer quickly by sorting the table.

____

10) What code would you write to answer your question?

__________ (__________ , __________ , __________)
function-name table-name :: Table column-name :: String order :: Boolean

Functions for Tables (continued)

count

```
# count :: Table, String -> Table
```

1) What is the Domain of `count` ? ______________________________

2) What is the Range of `count` ? ______________________________

3) What do you suspect the String in the Domain will describe? ______________________________

Suppose we wanted to know how many animals had 4 legs...
Type `count(animals-table, "legs")` into the Interactions Area and click "Enter"

4) What did the expression produce? ______________________________

5) How many animals had 4 legs? ______________________________

6) Think of another question you might be able to answer with the `count` function.

____

7) Fill in the blanks with the code you'd write.

__________ (__________________ , __________________)
function-name   table-name :: Table   column-name :: String

8) Tables that summarize data with a count are commonly used in the real world. Give two examples of where you've seen them before:

- Example 1: ______________________________
- Example 2: ______________________________

9) Newscasters and journalists often incorporate data into their reporting. How else might they display this information, besides using a table?

____

first-n-rows

10) Type `first-n-rows(animals-table, 5)` . What happens? ______________________________

11) If we wanted a table of the first 3 rows of the `animals-table` , what code would you write? ______________________________

12) What is the Contract for `first-n-rows` ? ______________________________

★ What happens when you type `first-n-rows(sort(animals-table, "pounds", true), 5)` ?

____

Note: In this case, the output of `sort(animals-table, "pounds", true)` *is the Table* `first-n-rows` *is taking in!*

★★ See if you can figure out how to compose the code that would generate a table of the 10 oldest animals!

__________ (______________________________ , __________)
function-name   Table   Number

Circles of Evaluation: Count, Sort, First-n-rows

For each scenario below, draw the Circle of Evaluation and then use it to write the code.
When you're done, test your code out in the Animals Starter File and make sure it does what you'd expect it to.

```
# count :: Table, String -> Table
# first-n-rows :: Table, Number -> Table
# sort :: Table, String, Boolean -> Table
```

1) **We want to see the 10 animals who were adopted the quickest.**
Circle of Evaluation:

code: ____________________

2) **We want to see the heaviest animal.**
Circle of Evaluation:

code: ____________________

3) **We want to take the first 8 animals from the table and put them in alphabetical order (by name).**
Circle of Evaluation:

code: ____________________

4) **You notice that the lightest 16 animals weigh under 10 pounds and you want to know the count (*by species*) of those animals.**
Circle of Evaluation:

code: ____________________

Catching Bugs when Sorting Tables

Learning about a Function through Error Messages

1) Type `sort` into the Interactions Area of the [Animals Starter File] and hit "Enter". What do you learn? ______________________

2) We know that all functions need an open parenthesis and at least one input! Type `sort(animals-table)` in the Interactions Area and hit Enter/return. Read the error message. What hint does it give us about how to use this function?

____

What Kind of Error is it?

syntax errors - when the computer cannot make sense of the code because of unclosed strings, missing commas or parentheses, etc.
contract errors - when the function isn't given what it needs (the wrong type or number of arguments are used)

3) In your own words, the difference between ***syntax errors*** and ***contract errors*** is: ______________________

____

Finding Mistakes with Error Messages

The code below is BUGGY! Read the code and the error messages, and see if you can catch the mistake WITHOUT typing the code into Pyret.

4) `sort(animals-table, name , true)`

```
The name name is unbound:
sort(animals-table, name , true)
It is used but not previously defined.
```

This is a ______________ error. The problem is that ______________________
contract / syntax

5) `sort(animals-table, "name" , "true")`

```
The Boolean annotation:
fun sort(t :: Table, col :: String, asc :: Boolean)
was not satisfied by the value
    "true"
```

This is a ______________ error. The problem is that ______________________
contract / syntax

6) `sort(animals-table "name" true)`

```
Pyret didn't understand your program around:
sort(animals-table "name" true)
You may need to add or remove some text to fix your program. Look carefully before the
highlighted text. Is there a missing colon (:), comma (,), string marker ("), or keyword? Is
there something there that shouldn't be?
```

This is a ______________ error. The problem is that ______________________
contract / syntax

7) `sort(animals-table, "name", true`

```
Pyret didn't expect your program to end as soon as it did:
sort(animals-table, "name", true
You may be missing an "end", or closing punctuation like ")" or "]" somewhere in your program.
```

This is a ______________ error. The problem is that ______________________
contract / syntax

8) `sort (animals-table, "name", true)`

```
Pyret thinks this code is probably a function call:
sort (animals-table, "name", true)
Function calls must not have space between the function expression and the arguments.
```

This is a ______________ error. The problem is that ______________________
contract / syntax

Contracts for Image-Producing Functions

Log into [code.pyret.org (CPO)](https://code.pyret.org) and click "Run". Experiment with each of the functions listed below, trying to find an expression that will build. Record the contract and example code for each function you are able to successfully build!

Name		Domain		Range
`# triangle`	`::`	`Number, String, String`	`->`	`Image`
`triangle(80, "solid", "darkgreen")`				
`# star`	`::`		`->`	
`# circle`	`::`		`->`	
`# rectangle`	`::`		`->`	
`# text`	`::`		`->`	
`# square`	`::`		`->`	
`# ellipse`	`::`		`->`	
`# regular-polygon`	`::`		`->`	

Challenge: Composing with Circles of Evaluation

What if we wanted to see your name written on a diagonal?

- We know that we can use the `text` function to make an Image of your name.
- Pyret also has a function called `rotate` that will rotate any Image a specified number of degrees.

```
# rotate :: Number, Image -> Image
```

But how could the `rotate` and `text` functions work together? Draw a Circle of Evaluation, translate it to code and test it out in the Editor!

Exploring Displays

Use the contracts provided below to make each type of display in the Animals Starter File. Then answer the questions about each display.

Bar Charts `# bar-chart :: Table, String -> Image`

________________ (______________________________ , ________________________)
function-name | table-name :: Table | column-name :: String

Sketch a bar chart below.

Bar charts summarize 1 column of ________________ data.
categorical/quantitative

This kind of display tells us...

Pie Charts `# pie-chart :: Table, String -> Image`

________________ (______________________________ , ________________________)
function-name | table-name :: Table | column-name :: String

Sketch a pie chart below.

Pie charts summarize 1 column of ________________ data.
categorical/quantitative

This kind of display tells us...

Box Plots `# box-plot :: Table, String -> Image`

________________ (______________________________ , ________________________)
function-name | table-name :: Table | column-name :: String

Sketch a box plot below.

Box plots summarize 1 column of ________________ data.
categorical/quantitative

This kind of display tells us...

Histograms `# histogram :: Table, String, String, Number -> Image`

________________ (________________________ , ________________ , ________________ , ________________)
function-name | table-name :: Table | labels :: String | values :: String | bin-width :: Number

Sketch a histogram below.

Histograms summarize 1 column of ________________ data.
categorical/quantitative

This kind of display tells us...

Circles of Evaluation: Composing Functions to Make Displays

Using the Contracts below as a reference, draw the Circle of Evaluation for each prompt.

```
# pie-chart :: Table, String -> Image
# bar-chart :: Table, String -> Image
# histogram :: Table, String, String, Number -> Image
# box-plot :: Table, String -> Image
# first-n-rows :: Table, Number -> Table
# sort :: Table, String, Boolean -> Table
```

1) Make a bar-chart of the lightest 16 animals by `sex`.

★ What other bar chart might you want to compare this to? ____________________

2) Take the heaviest 20 animals and make a histogram of `weeks` to adoption (use `"species"` for your labels).

★ What other histogram might you want to compare this to? ____________________

3) Make a box-plot of `age` for the 11 animals who spent the most `weeks` in the shelter.

★ What other box plot might you want to compare this to? ____________________

4) Make a pie-chart of `species` for the 18 animals who spent the fewest `weeks` in the shelter.

★ What other pie chart might you want to compare this to? ____________________

Exploring the States Dataset

Open the Preview: State Demographics Starter File.
Then, click "Run" and type `states-table` into the Interactions Area on the right to see the dataset.

What do you Notice about this dataset?	What do you Wonder about this dataset?

1) What code will produce a table showing the number of states in each region? ______

2) Which states do you **think** have the most people? ______

3) What code will produce a table containing the five states with the largest population in 2020?

______

4) Which states do you **think** have the most poverty? ______

5) What code will produce a table containing the ten states with the highest % of people in poverty?

______

6) What code will produce a table containing the states with the lowest **median** income?

______

7) What code will produce a table containing the states with the lowest **per-capita** ("average" or "mean") income?

______

★ What does it mean if a state has a higher `per-capita income` than `median-income`? ______

______

The two lines of code under `# Define some rows` *extract rows 0 and 1 from the table, and define them as* `alabama` *and* `alaska`.

8) Type `alabama` into the Interactions Area. What do you get back? ______

9) Underneath the definition of those rows, **add a new definition** for `california` and click "Run", so that Pyret reads your new definition.

10) Add a definition for your own state, then **click "Run"** and test it out in the Interactions Area!

11) Add any additional Notices or Wonderings you have about this dataset to the table at the top.

Looking for Patterns

Open the [Preview: State Demographics Starter File](#).

Part 1

1) What columns do you think might be related to one another? (e.g. - is the number of veterans related to the amount of land-area? Is the population in 2010 related to the population in 2020?) List three possible relationships below.

a. I think that ______________________ may be related to ______________________

b. I think that ______________________ may be related to ______________________

c. I think that ______________________ may be related to ______________________

```
# scatter-plot :: (Table, String, String, String) -> Image
                          labels  explanatory response
```

2) Use the Contract above to make a scatter-plot for the **first relationship** you wrote above. (Use `"state"` as the label, so that clicking on a point will show you which state you're looking at.)

a. If there's a pattern in this scatter-plot, what does that mean? If there isn't, what does *that* mean? ______________________

____

b. In your own words, describe the pattern you see in the scatter plot so someone else could sketch it. ______________________

____

____

3) Make a scatter-plot for the **second relationship** you wrote.

a. If there's a pattern in this scatter-plot, what does that mean? If there isn't, what does *that* mean? ______________________

____

b. In your own words, describe the pattern you see in the scatter plot so someone else could sketch it. ______________________

____

____

4) Make a scatter-plot for the **third relationship** you wrote.

a. If there's a pattern in this scatter-plot, what does that mean? If there isn't, what does *that* mean? ______________________

____

b. In your own words, describe the pattern you see in the scatter plot so someone else could sketch it. ______________________

____

____

Part 2

Wait to complete this until after diving deeper into statistical relationships!

Revisit the three scatter plots you made and add the following labels to the descriptions you wrote in Question 1:

- Place an "L" by any relationships that you think might be linear.
- Place a "P" by any relationships that appear to be positive.
- Place an "N" by any relationships that appear to be negative.
- Place an "S" by the strongest-looking relationship.
- Place a "W" by the weakest-looking relationship.

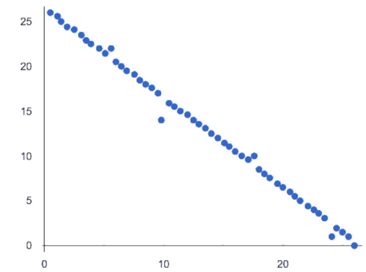
Identifying Form, Direction and Strength (Matching)

Match the description (left) with the scatter plot (right).

Note: The computer won't tell us if the relationship we see is linear, so we must train our eyes to decide this ourselves. For linear relationships, we should train our eyes to assess their direction and get a feel for their strength, rather than relying completely on what numbers the computer reports.

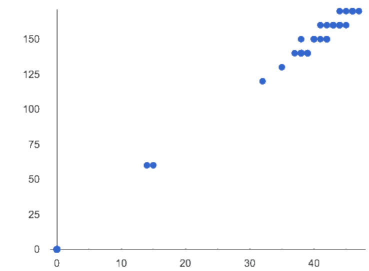
The relationship appears to be linear, negative, and of moderate strength. **1**

A



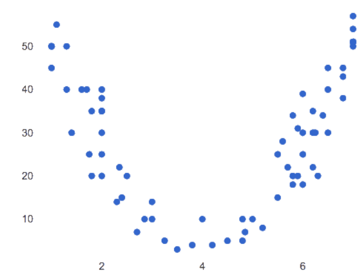
This relationship is nonlinear. **2**

B



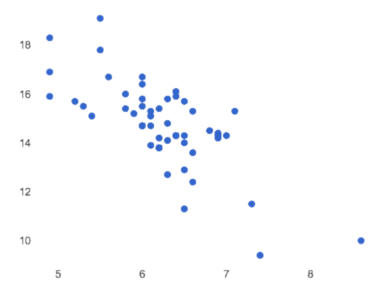
The x and y variables in this dataset do not appear to be related. **3**

C



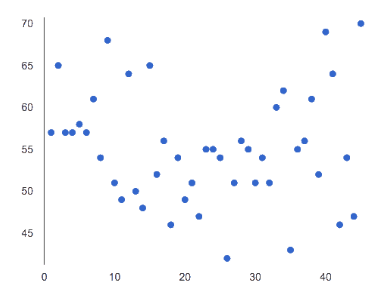
The relationship appears to be linear, positive, and strong. **4**

D



The relationship appears to be linear, negative, and strong. **5**

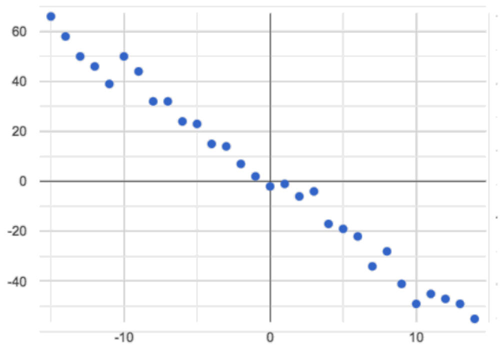
E



Identifying Form, Direction and Strength

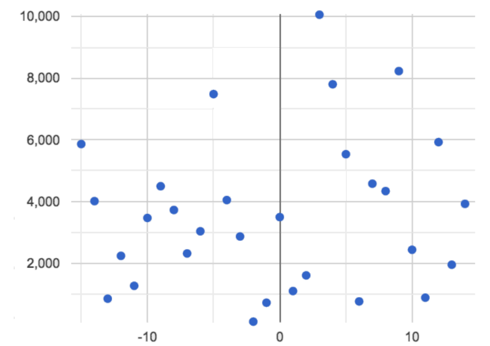
What do your eyes tell you about the Form, Direction, & Strength of these displays?
Note: If the form is nonlinear, we shouldn't report direction - a curve may rise and then fall.

A



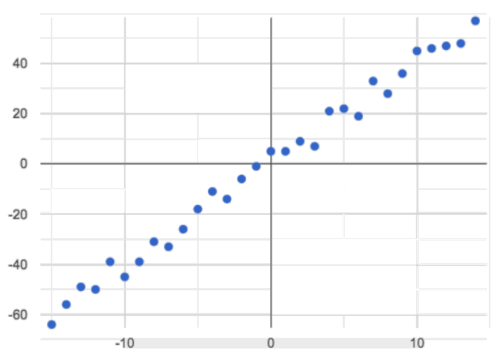
Form: Linear Nonlinear None
Direction: Positive Negative N/A
Strength: Strong Weak

B



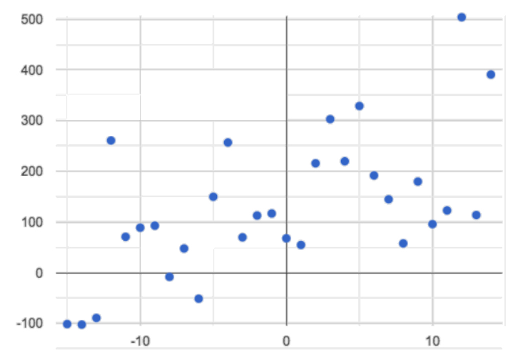
Form: Linear Nonlinear None
Direction: Positive Negative N/A
Strength: Strong Weak

C



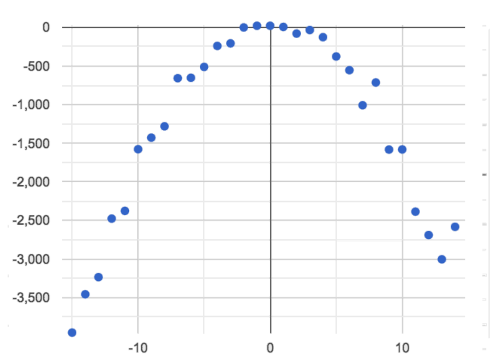
Form: Linear Nonlinear None
Direction: Positive Negative N/A
Strength: Strong Weak

D



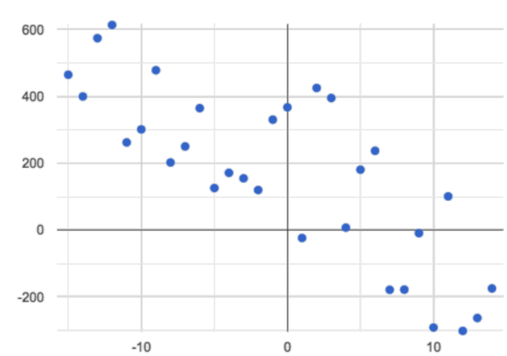
Form: Linear Nonlinear None
Direction: Positive Negative N/A
Strength: Strong Weak

E



Form: Linear Nonlinear None
Direction: Positive Negative N/A
Strength: Strong Weak

F



Form: Linear Nonlinear None
Direction: Positive Negative N/A
Strength: Strong Weak

Build a Model from Samples: College Degrees v. Income

Open the [Preview: State Demographics Starter File](#).

1) Record the `pct-college-or-higher` and `median-income` values for the `alabama` and `alaska` rows, as (x,y) pairs below:

(________________, ________________)
AL pct-college-or-higher, AL median-income

(________________, ________________)
AK pct-college-or-higher, AK median-income

2) Using the space below, compute the equation of the line passing between these two points. **This line will be your linear model** (also known as the "predictor function", or "line of best fit"), which predicts `median-income` as a function of `pct-college-or-higher`.

3) Write the complete model below (in both Function and Pyret notation):

$\text{al-ak}(x) =$ ____________ x + ________________
slope(m) — y-intercept / vertical shift

`fun al-ak(x): (____________ * x) + ____________ end`

Return to your copy of the starter file and add the code you just wrote to the Definitions Area. Then Click "Run".
(If there are any errors or warnings, fix them and click "Run" again.)

4) In the Interactions Area, try plugging in the `pct-college-or-higher` value for Alabama by typing `al-ak(22.6)`.

- How well does it predict the correct median income for Alabama? ____________________________
- What expression would predict median income for Alaska? ____________________________
- How well does it predict the correct median income for Alaska? ____________________________
  Consider: If it doesn't predict it perfectly, why might that be?

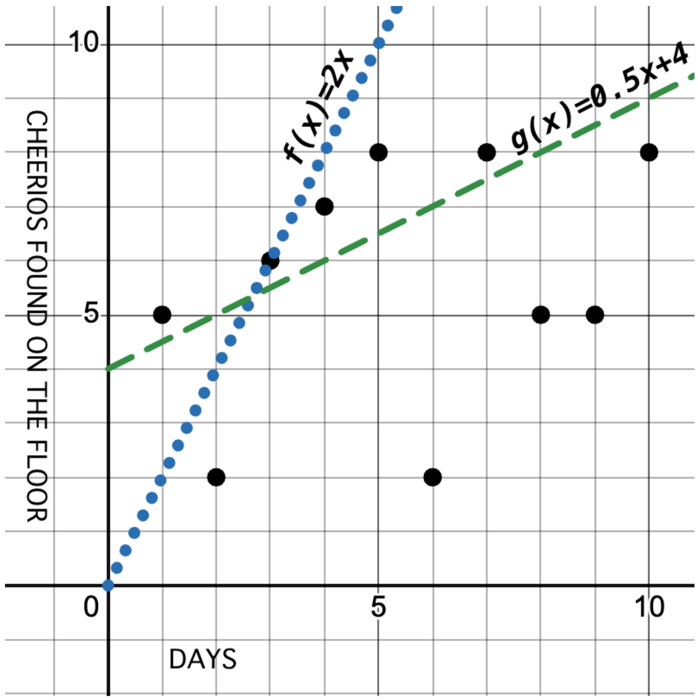
Try different `pct-college-or-higher` values from *other* states, to see how well our Alabama-Alaska model fits the rest of the country.

5) Identify a state for which this model works well: ____________________________

6) Identify a state for which this model works poorly: ____________________________

7) What median income does this model expect a state without ANY college graduates (0%) to earn? __________________

How could we Measure Whether a Model is a Good Fit?



id	Days	Cheerios found on the floor
a	1	5
b	2	2
c	3	6
d	4	7
e	5	8
f	6	2
g	7	8
h	8	5
i	9	5
j	10	8

1) Do you think $f(x)$ or $g(x)$ is a better model for this data? ______________________

2) What makes you think that? ______________________

3) What could we measure, to calculate *how much better of a model* it is? ______________________

4) *Neither of these models is the best possible model!* What would have to be true of a third model, for us to know that it was a better fit than these two? ______________________

Fit a Model: College Degrees v. Income

*Open the [Fitting a Model: State Demographics Starter File] and **Save a Copy** of the file that's just for you.*

The `al-ak` Model

Type `fit-model(states-table, "state", "pct-college-or-higher", "median-income", al-ak)` in the Interactions Area, then find the points for AL and AK along the predictor line. *Hint: You know their coordinates and they will help you know where to look!*

1) What do you Notice?

____

____

2) What do you Wonder?

____

____

3) Find S in the upper left corner. What is the S value (the number after S)? ______________________

Other Models

In the definitions area, find the section titled *Define some other models by modifying al-ak* .

- For now, all three definitions in this section are exactly the same as `al-ak` .
- You will be changing them according to the directions below.

4) If you wanted the model to be *less steep* , what slope could you use? ______________________

- Change the definition for `less-steep` to use the slope you wrote above.
  - Click "Run" to load your new definition. In the Interactions Area type: `fit-model(states-table, "state", "pct-college-or-higher", "median-income", less-steep)`
  - What is the S value of `less-steep` ? ______________________
- Identify a y-intercept that would make the model fit the data better: ______________________
  - Adjust the definition to use the new y-intercept and click "Run".
  - Hit the up arrow in the Interactions Area and click return/Enter to fit the model again.
  - What is the S value of `less-steep` now? ______________________

5) Change the definition of `negative` so that it models the data with a slope that is *negative* .

- Click "Run" and type the code to fit this model to the data.
- What slope did you use? ____________ What is the S value now? ______________________

6) Change the definition of `horizontal` so that it draws a horizontal model. Click "Run" and fit this model. What is the S value? ________

7) Change the y-intercept so that the horizontal line passes through more of the points. Click "Run" and fit this model.

- What y-intercept did you use? ____________ What is the S value now? ______________________

8) What do you think S tells us? ______________________

____

____

____

What does *S* tell us about the fit of these models?

For each model below, decide whether the fit is "poor", "ok", or "good". *Then* rank the models from 1 (best fit) to 8 (worst fit).

How good is the model?	Ranking
1 A data scientist is working with data from animals at a shelter. • The range of days to adoption in this dataset are from 0 to 400. • An S value of 300 means predicted adoption times could be off by 300 days. This is a(n) ______________________ model for the dataset. poor, ok, good	
2 A student is exploring a dataset on climate change. • The range of Arctic Sea Ice is from 3,920,000 to 7,670,000 square kilometers • An S value of 300 means predicted Arctic Sea Ice coverage could be off by 300 square kilometers. This is a(n) ______________________ model for the dataset. poor, ok, good	
3 A data scientist is working with data from US public schools. • The range of graduates per school per year is 2 to 2003. • An S value of 300 means predicted graduate values could be off by 300 students. This is a(n) ______________________ model for the dataset. poor, ok, good	
4 A student is exploring a dataset on earthquakes. • The range of earthquake depths in this dataset are from 4200m to 664000m. • An S value of 300 means predicted earthquake depths could be off by 300 meters. This is a(n) ______________________ model for the dataset. poor, ok, good	
5 A student is exploring a dataset on arrests in Los Angeles. • The age range in this dataset is from 0 to 92. • An S value of 1 means predicted ages could be off by 1 year. This is a(n) ______________________ model for the dataset. poor, ok, good	
6 A data scientist is working with data about snowflakes. • The range of snowflake weights is from 0.001 grams to 0.02 grams. • An S value of 1 means predicted values could be off by 1 gram. This is a(n) ______________________ model for the dataset. poor, ok, good	
7 A data scientist is working with data from animals at a shelter. • The range of ages is from 0.5 years to 16 years. • An S value of 1 means predicted ages could be off by 1 year. This is a(n) ______________________ model for the dataset. poor, ok, good	
8 A student is working with a dataset of adult blue whales. • The range of weights is 200,000 to 330,000 pounds. • An S value of 1 means predicted weights could be off by 1 pound. This is a(n) ______________________ model for the dataset. poor, ok, good	

Better Modeling: College Degrees v. Income

Open your copy of the [Fitting a Model: State Demographics Starter File](#).

Build a Model through Trial & Error

Find `# Define some rows` in the Definitions Area.
Add two new definitions for `MA` (row 21) and `NV` (row 28), using the definitions for `alaska` and `alabama` as a model.

1) Record the `college-or-higher` and `median-income` values for `MA` and `NV`, as (x,y) pairs below:

(________________, ________________)
MA college-or-higher, MA median-income

(________________, ________________)
NV college-or-higher, NV median-income

2) Derive the `MA–NV model` (using the same steps you followed to derive the `AL–AK model` on [Fit a Model: College Degrees v. Income](#)) and write it below (in both Function and Pyret notation), then fit the model and record the S-value:

ma - $nv(x) =$ ____________ x + ________________
slope (m), y-intercept / vertical shift

`fun ma-nv(x): ( ________ * x) + ________ end`

S: ________

3) Identify two other states that you think would make a better model: ____________________ and ____________________.

Add two new definitions for these states to your [Fitting a Model: State Demographics Starter File](#).

4) Record the `college-or-higher` and `median-income` values for these states, as (x,y) pairs below:

(________________, ________________)
college-or-higher, median-income

(________________, ________________)
college-or-higher, median-income

5) Derive your model and write it below (in both Function and Pyret notation), then fit the model and record the S-value:

$other(x) =$ ____________ x + ________________
slope (m), y-intercept / vertical shift

`fun other(x): ( ________ * x) + ________ end`

S: ________

6) Adjust the slope and y-intercept of your model to get the **smallest** S **possible.** Write the best model you find (and corresponding S) below:

$best(x) =$ ____________ x + ________________
slope (m), y-intercept / vertical shift

`fun best(x): ( ________ * x) + ________ end`

S: ________

Optimizing and Interpreting Linear Models

Open your copy of the [Fitting a Model: State Demographics Starter File](#).

Build a Model Computationally

`lr-plot` computes the *optimal linear model* using all of the data points.

1) Evaluate `lr-plot(states-table, "state", "pct-college-or-higher", "median-income")`. What is S? ____________

2) On the line below, write the optimal linear model that was computed through linear regression:

$optimal(x) =$ ______________ (slope (m)) x + ______________ (y-intercept / vertical shift)

`fun optimal(x): ( ____________ * x) + ____________ end`

Interpret the Model

We started with a model based on Alabama and Alaska `fun al-ak(x): (5613.67 * x) + -83616.02 end` `S: ~36164.68` which we can interpret as follows:

The *Alabama-Alaska* (sensible name) *model predicts that a 1* *percent* (x-axis units) *increase in* *percent college degrees* (x-axis) *is associated with a* *5613 dollar* (slope, y-units) *increase* (increase / decrease) *in* *median household income* (y-axis). *With an* S *-* $value$ *of* *~36,164.68* (S-value) *dollars* (y-units) *and* *median household income* (y-axis) *ranging from* *$39,031* (lowest y-value) *to* *$73,538* (highest y-value), *this model fits* *really, really poorly* (really well, decently, poorly, etc.).

3) Describe the optimal model YOU created via linear regression:

The linear-regression (sensible name) model predicts that a 1 ______________ (x-axis units) increase in ______________ (x-axis) is associated with a ______________ (slope, y-units) ______________ (increase / decrease) in ______________ (y-axis). With an S-value of ______________ (S-value) dollars (y-units) and ______________ (y-axis) ranging from ____________ (lowest y-value) to ____________ (highest y-value), this model fits ______________ (really well, decently, poorly, etc.).

4) What does the **slope (m)** of this linear model tell us? ______________________________

____

5) What does the **y-intercept / vertical shift** of this linear model tell us? ______________________________

____

6) Suppose a state goes from 10% to 11% college graduation. According to this model,

- What kind of change would we expect to see in the median household income? ____________
- What if it goes from 50% to 51%? ______________________________
- What if it goes from 90% to 91%? ______________________________

7) Does this model predict the same increase in income for *every* additional 1% `college-or-higher`? Why or why not? ____________

____

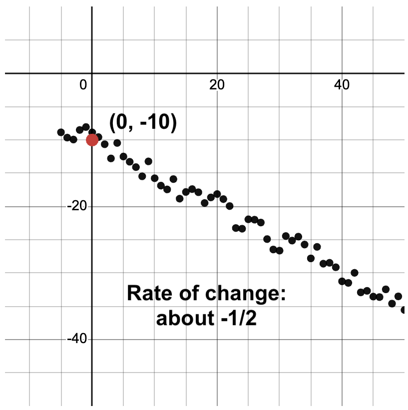
____

Which Form is Best?

For each set of data provided below,

- Decide which form of the line would be the easiest to build from the available information.
- Write a definition of the linear model in that form.
- Translate the definition into Pyret notation.

1

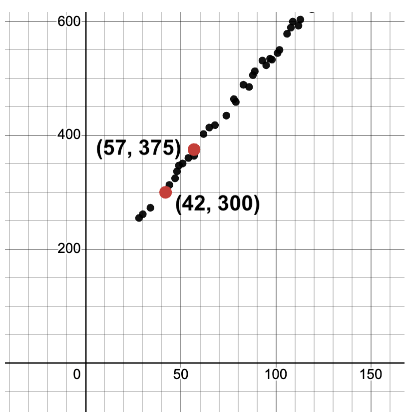


Linear Model: ______________________________

Your model slope-intercept, point-slope, or standard form - which ever is easiest!

`fun f(x) :` ______________________________ `end`

2

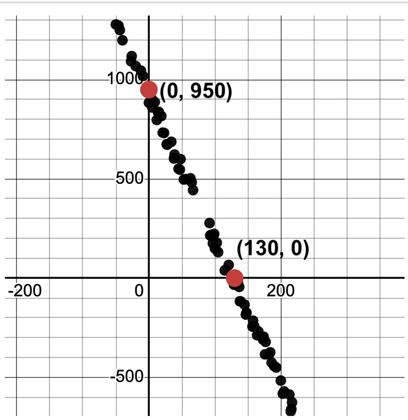


Linear Model: ______________________________

Your model slope-intercept, point-slope, or standard form - which ever is easiest!

`fun f(x) :` ______________________________ `end`

3



Linear Model: ______________________________

Your model slope-intercept, point-slope, or standard form - which ever is easiest!

`fun f(x) :` ______________________________ `end`

Exploring the Fuel Efficiency Dataset

*For this page, you'll need to open the [Fuel Efficiency Starter File] on your computer. If you haven't already, select **Save a Copy** from the "File" menu to make a copy of the file that's just for you. **Read the comments at the top of the file**, which describe what each column in the dataset means.*

Fitting Linear Models

1) Evaluate `A15`, `A45` and `A75` in the Interactions Area. What **model** of car is used in all three rows? ______________________

2) At what three **speeds** is this model being tested in these rows? ______________________

3) Does there appear to be a relationship between speed and miles-per-gallon? __________.

4) *Looking at the numbers in the* `mpg-table`, describe its **form** (e.g. - linear, non-linear, or none) and **strength** (strong, moderate, or weak). If it appears to be linear, what is the **direction**? If it does *not* appear to be linear, describe its shape.

____

____

5) Use `lr-plot(mpg-table, "model", "speed", "mpg")` to find the optimal **linear** model. What is S for this model? __________
Write the model below, in both math and Pyret notation.

$y =$ ______________ x + ______________
slope — y-intercept / vertical shift

`fun y(x): ( ______________ * x) + ______________ end`

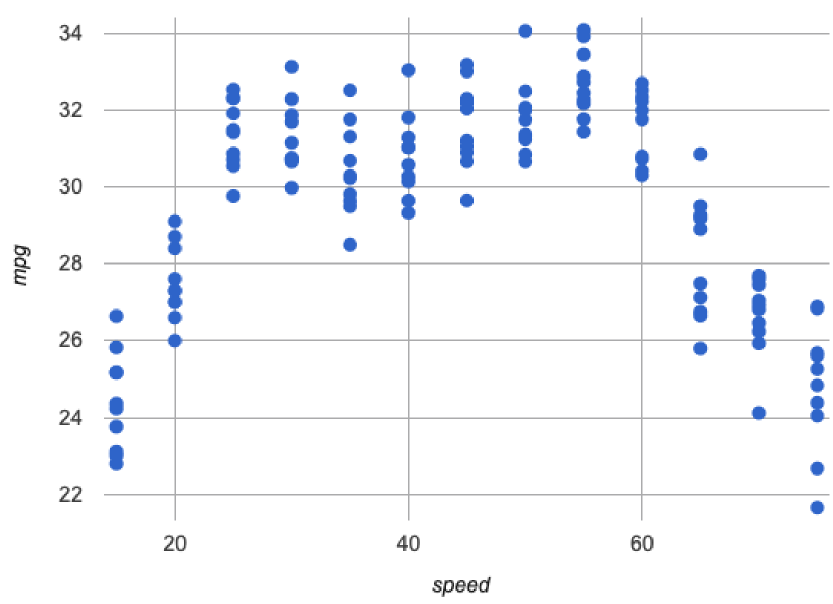
6) Is the best-possible linear model a good fit? __________. Why or why not? ______________________

____

____

Fitting Curves

7) Sketch your lr-plot in the space below, showing the relationship between `speed` and `mpg`. Be sure to label your axes, and draw the linear model!



8) What do you **Notice?** ______________________

____

____

____

9) What do you **Wonder?** ______________________

____

____

____

10) Do you think a **curve** would fit better?

____

11) Draw a **curve** on your scatter-plot, which shows the overall shape in the data. At what speed is the "peak"? ______________

12) Based on your best-guess curve, what do you predict `mpg` would be for a new test run at 25mph ______? 60mph ______? 90mph ______?

What Kind of Model? (Descriptions)

Decide whether each situation sounds like it would be better modeled by a linear or quadratic function, and circle your answer.

1) A car is 50 miles away, traveling at 65mph. How far away is the car after each hour?

Linear Quadratic

---

2) A ball is dropped from the top of the Empire State Building, and it keeps dropping faster and faster. **How far has the ball dropped** after x seconds?

Linear Quadratic

---

3) The data plan for a cell phone bill costs $5/gb, plus $15/mo. How much is the bill for a given month, after x number of gigabytes?

Linear Quadratic

---

4) A ball is dropped from the top of the Empire State Building, and it keeps dropping faster and faster. **How fast is the ball moving** after x seconds?

Linear Quadratic

---

5) A cannonball is fired from the deck of the S.S. Parabola, and arcs through the sky before hitting its target, 17 miles away.

Linear Quadratic

---

6) The area of a circle, as its radius increases.

Linear Quadratic

---

7) The circumference of a circle, as its radius increases.

Linear Quadratic

What Kind of Model? (Tables)

Decide whether each representation is best described by a linear model, a quadratic model or neither! Show any work that you feel is useful.

For Class Discussion:

1

x	0	1	2	3	4	5	6
y	5	6	9	14	21	30	41

Linear
Quadratic
Neither

2

x	0	1	2	3	4	5	6
y	0	3	6	9	12	15	18

Linear
Quadratic
Neither

For Independent Practice:

3

x	1	2	3	4	5	6	7
y	1	3	5	7	9	11	13

Linear
Quadratic
Neither

4

x	-3	-2	-1	0	1	2	3
y	-23	-38	-47	-50	-47	-38	-23

Linear
Quadratic
Neither

5

x	-3	-2	-1	0	1	2	3
y	1	2	1	2	1	1	1

Linear
Quadratic
Neither

6

x	1	2	3	4	5	6	7
y	2	5	10	17	26	37	50

Linear
Quadratic
Neither

7

x	-3	-2	-1	0	1	2	3
y	12	7	2	-3	-8	-13	-18

Linear
Quadratic
Neither

8

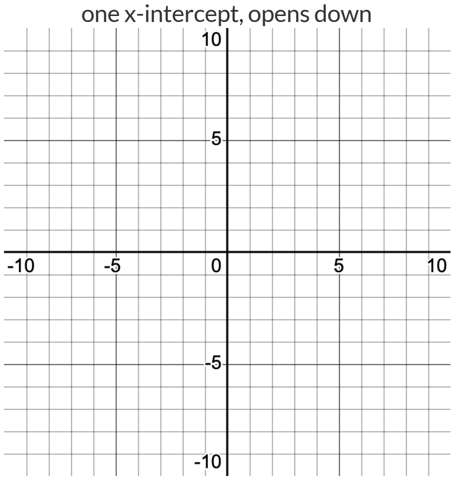
x	1	2	3	4	5	6	7
y	100	102	105	109	114	120	127

Linear
Quadratic
Neither

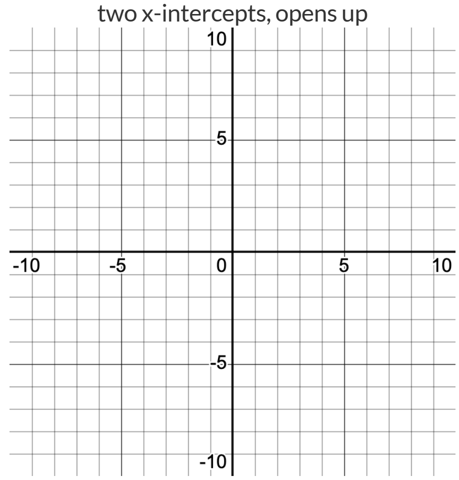
Parabolas

1) Sketch a ***parabola*** on each of the grids below that matches the description.

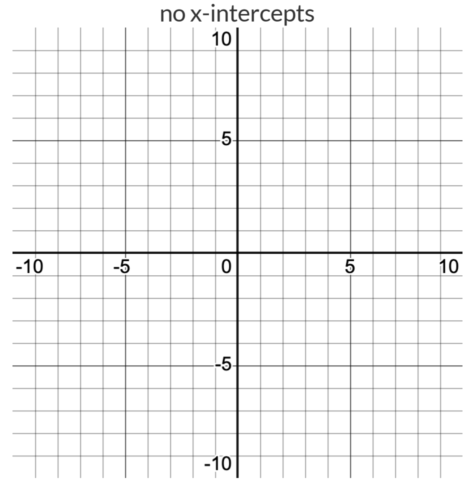
one x-intercept, opens down



two x-intercepts, opens up



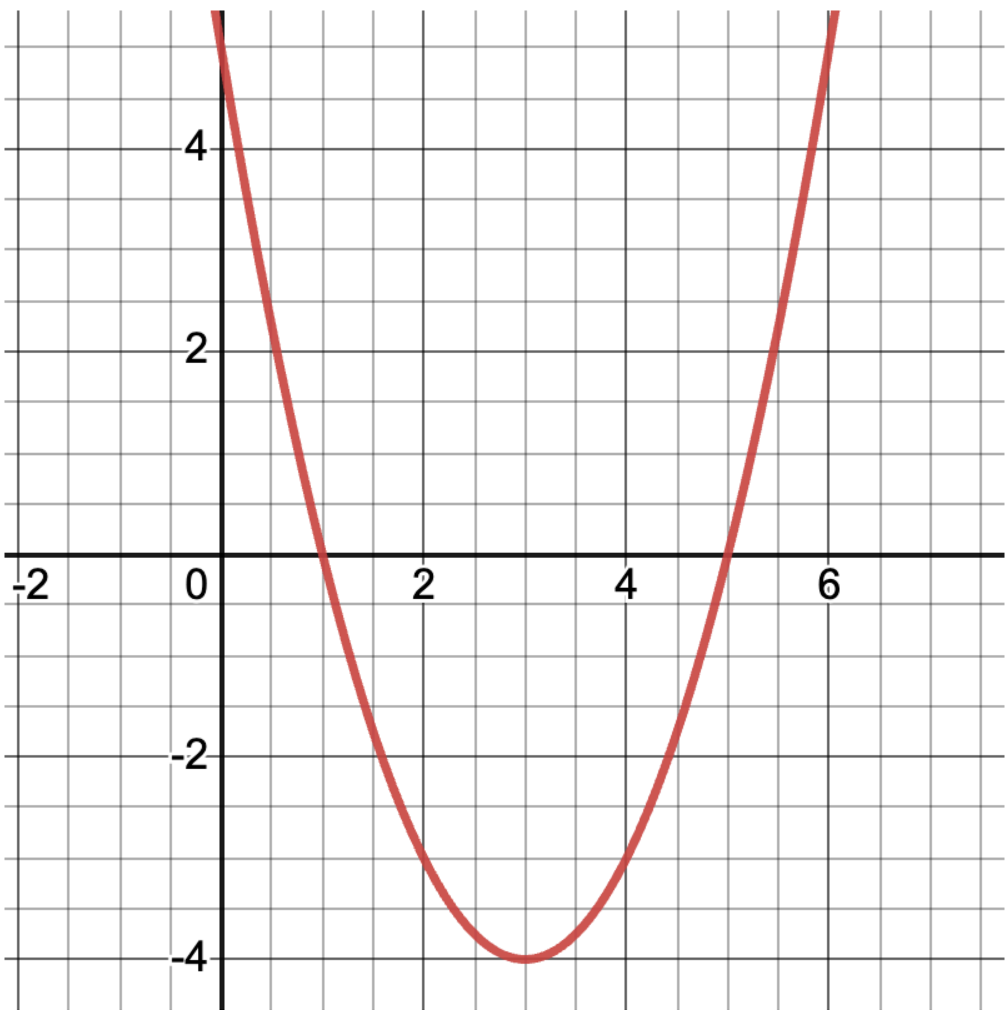
no x-intercepts



2) Label the ***vertex***, ***root(s)***, and ***y-intercept*** of the parabola below with:

A) their coordinates

B) the vocabulary word (above) that describes each



3) Draw a dotted line representing the ***axis of symmetry*** and label it with the equation that defines it.

Graphing Quadratic Models

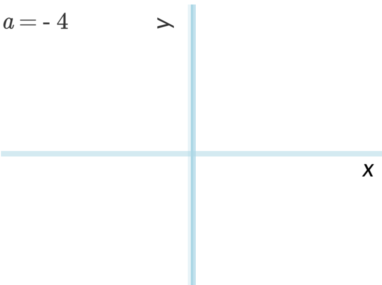
For this page, you'll need to have ***Exploring Quadratic Functions(Desmos)*** *open on your computer.*
The parabola you'll see is the graph of $g(x) = x^2$. Another, **identical** parabola is hiding behind it.
This second parabola is written in Vertex Form: $f(x) = a(x - h)^2 + k$. Each coefficient starts at values to make $f(x)$ equivalent to $g(x)$.

1) Using the starting values of a, h, and k you see for $f(x)$ in Desmos, rewrite $g(x) = x^2$ in Vertex Form. $g(x) =$ ____________________

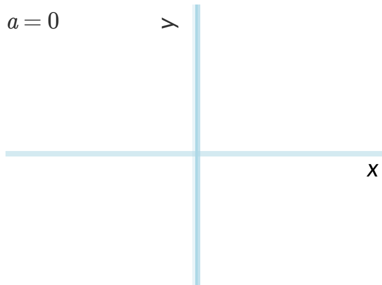
Magnitude a

2) Try changing the value of a to -4, 0, and 2, graphing each parabola in the squares below. **Label the vertex "V" and any roots with "R"!**

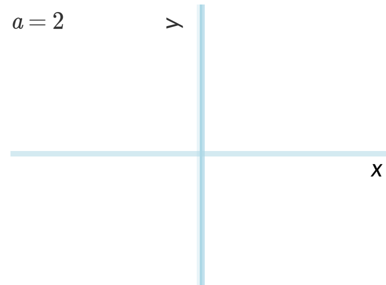
$a = -4$



$a = 0$



$a = 2$



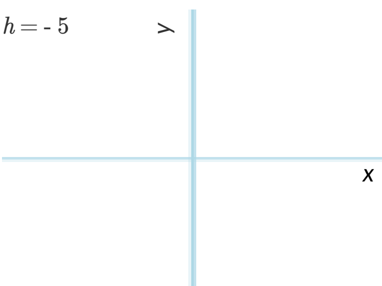
3) What does a tell us about a parabola? ____

____

Horizontal Translation h

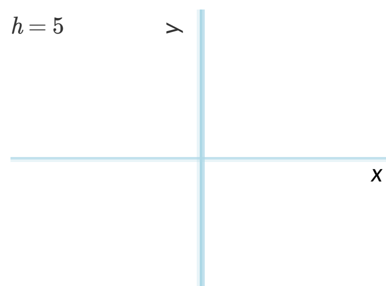
4) Set a back to 1. Change the value of h to -5, 0, and 5, graphing each parabola in the squares below. **Label the vertex "V" and any roots "R"!**

$h = -5$



$h = 0$

$h = 5$

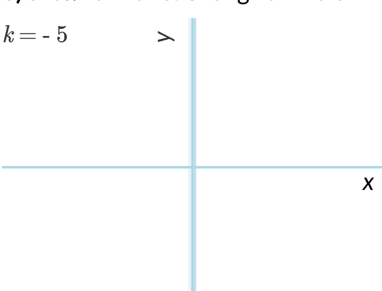


5) What does h tell us about a parabola? ____

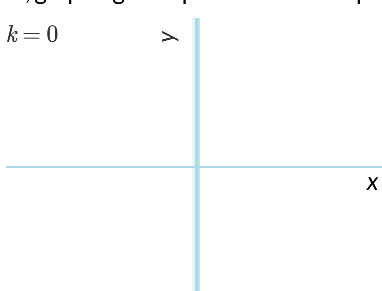
Vertical Translation k

6) Set h back to 0. Change the value of k to -5, 0, and 5, graphing each parabola in the squares below. **Label the vertex "V" and any roots "R"!**

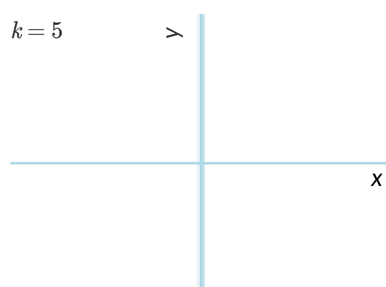
$k = -5$



$k = 0$



$k = 5$



7) What does k tell us about a parabola? ____

Modeling Fuel Efficiency v. Speed

Open your copy of the Fuel Efficiency Starter File and click "Run".

num-sqr

Before we try to model our fuel-efficiency data, we need to learn a new Pyret function!

1) Can you predict what the output of the `num-sqr` expressions below will be?

Test them out in the Interactions Area, and record the results. `num-sqr(4)` ______________ `num-sqr(6 - 2)` ______________

2) What is the Contract for `num-sqr` ? ____

3) What does `num-sqr` do? ____

Interpreting a Quadratic Model

In the Definitions Area of your Fuel Efficiency Starter File, you'll find the definition of a quadratic model `quad1` .

4) In `quad1` , the value of a is ____________________, the value of h is ____________________, and the value of k is ____________________

5) Fit this model to your dataset, using `fit-model` . What S-value did you get? ____

Hint: If you forgot the contract for `fit-model` *, look it up in the contracts pages!*

6) In your own words, describe what needs to change about this model to fit the data. ____

____

Modeling Fuel Efficiency

Vertex Form: $f(x) = a(x - h)^2 + k$	• a: determines whether the parabola opens up or down and how steep the curve is • h: horizontal shift (also the x-coordinate of the vertex! h is often 0) • k: vertical shift (also the y-coordinate of the vertex!)

7) We've determined that peak fuel efficiency is around 45 mph. What variable in the equation should we replace with 45? ______________

Update the definition of `quad1` , click "Run" and re-fit the model. What S-value did you get? ______________________________

8) What y-coordinate of the vertex (*vertical shift*) would best match the shape of the curve? __________

Update the definition of `quad1` , click "Run" and re-fit the model. What S-value did you get? ______________________________

9) What value of a best matches the shape of the curve? __________

Update the definition of `quad1` , click "Run" and re-fit the model. What S-value did you get? ______________________________

10) Make any small changes you'd like, trying to get S as low as you can. Write your final definition below.

`fun` `f(x) :` ____ **`end`** S: ______________________

What does this model actually mean?

After experimenting, I came up with a quadratic model for this dataset showing that __________ (x-variable) is correlated to ____________________ (y-variable). The error in the model is described by an ***S-value*** of about ______ (S) ______ (units), which is ______________________________ (insignificant, moderate, significant, extreme) considering that ____________________ (y-variable) in this dataset range from __________ (lowest y-value) to __________ (highest y-value). The vertex of the parabola drawn by this model is a ______________ (minima or maxima?) at about ________ (x, y) which means that ____

Before this point, as speed increases, `mpg` ________________. *After* this point, as speed increases `mpg` ________________

What Kind of Model? (Definitions)

Decide whether each representation describes a **linear** function, a **quadratic** function, or neither. If the function is quadratic, identify whether the **form** used is Vertex, Standard, or Factored.

$$f(x) = 3x^2 + 22$$

1) Linear Quadratic Neither

If Quadratic, is it Vertex, Standard, or Factored?

If Quadratic, what does the form tell you?

$$g(x) = 2(x - 11)(x - 243)$$

2) Linear Quadratic Neither

If Quadratic, is it Vertex, Standard, or Factored?

If Quadratic, what does the form tell you?

$$h(y) = 100 - 4y$$

3) Linear Quadratic Neither

If Quadratic, is it Vertex, Standard, or Factored?

If Quadratic, what does the form tell you?

$$z(x) = \frac{3}{5}x + 7$$

4) Linear Quadratic Neither

If Quadratic, is it Vertex, Standard, or Factored?

If Quadratic, what does the form tell you?

```
fun graph(x): 12 * x end
```

5) Linear Quadratic Neither

If Quadratic, is it Vertex, Standard, or Factored?

If Quadratic, what does the form tell you?

```
fun m(p): 2 * ((p - 5) * (p - 16)) end
```

6) Linear Quadratic Neither

If Quadratic, is it Vertex, Standard, or Factored?

If Quadratic, what does the form tell you?

$$r(s) = 42(s - 10)^2 - 3$$

7) Linear Quadratic Neither

If Quadratic, is it Vertex, Standard, or Factored?

If Quadratic, what does the form tell you?

```
fun f(x): (2 * num-sqr(x - 1)) + 15 end
```

8) Linear Quadratic Neither

If Quadratic, is it Vertex, Standard, or Factored?

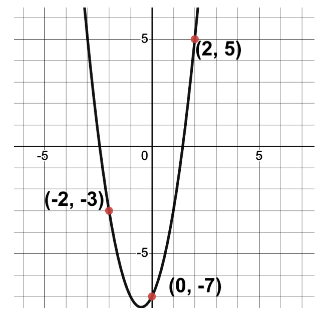
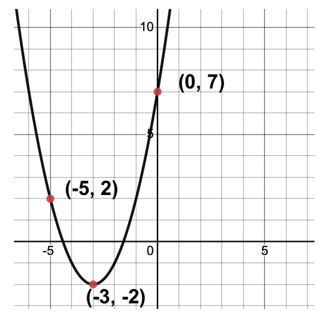
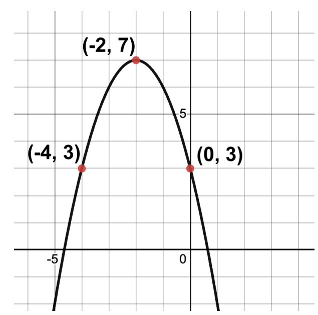
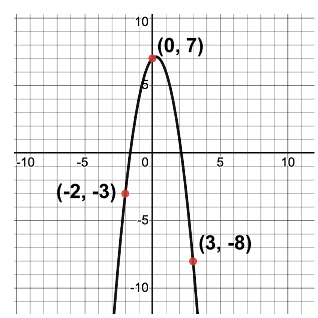
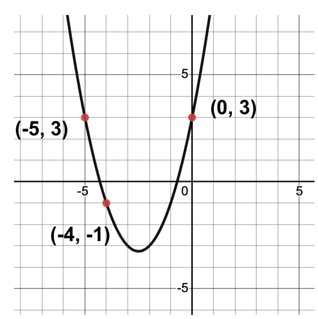
If Quadratic, what does the form tell you?

Matching Standard Form to Parabolas

Factored Form: $y = ax^2 + bx + c$

- a: determines whether the parabola opens up or down and how steep the curve is
- c: y-intercept

Match each definition below to the graph it describes.

$y = -x^2 - 4x - 3$	1	A	
$y = 2x^2 + 2x - 7$	2	B	
$y = x^2 + 5x + 3$	3	C	
$y = x^2 + 6x + 7$	4	D	
$y = -2x^2 + x + 7$	5	E	

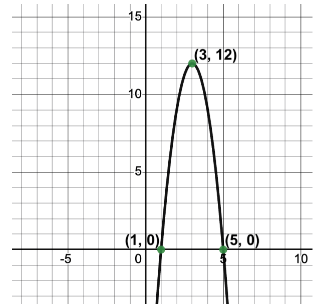
Matching Factored Form to Graphs

Factored Form: $y = a(x - r_1)(x - r_2)$

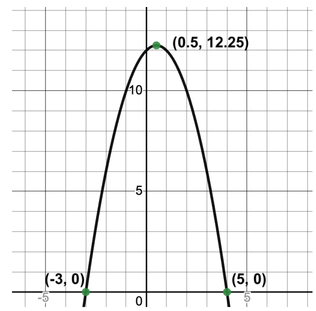
- a: determines whether the parabola opens up or down and how steep the curve is
- r_1 and r_2: roots, x-intercepts

Match each definition below to the graph it describes.

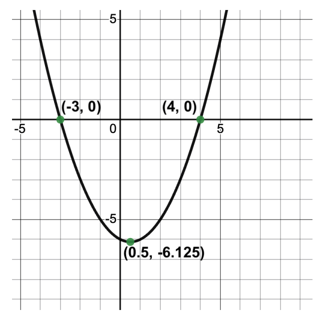
$y = 2(x - 1)(x + 5)$ **1** **A**



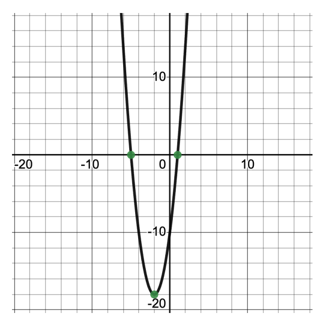
$y = (x + 3)(x + 4)$ **2** **B**



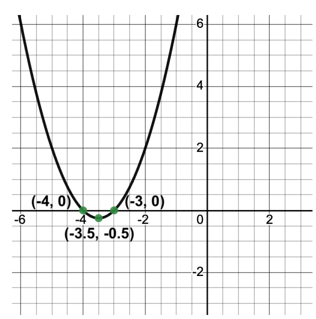
$y = -3(x - 1)(x - 5)$ **3** **C**



$y = \dfrac{1}{2}(x + 3)(x - 4)$ **4** **D**



$y = -(x - 5)(x + 3)$ **5** **E**



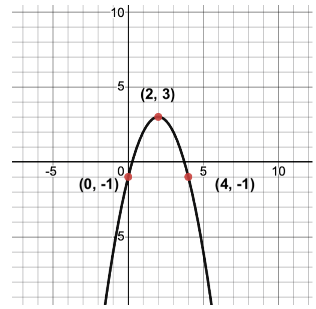
Matching Vertex Form to Graphs

Vertex Form: $y = a(x - h)^2 + k$

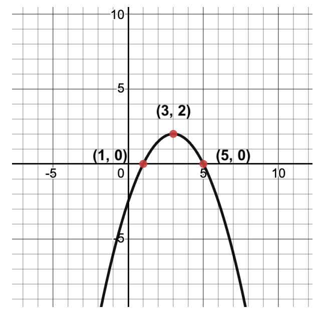
- a: determines whether the parabola opens up or down and how steep the curve is
- h: x-coordinate of the vertex
- k: y-coordinate of the vertex

Match each definition below to the graph it describes.

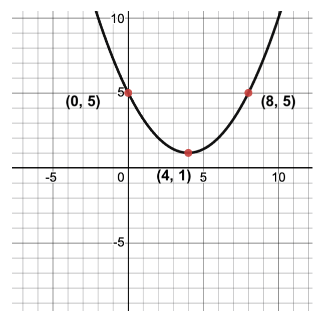
$f(x) = -\ 0.5(x - 3)^2 + 2$ 1 A



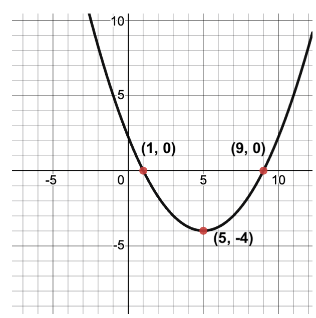
$g(x) = 2(x + 1)^2 - 4$ 2 B



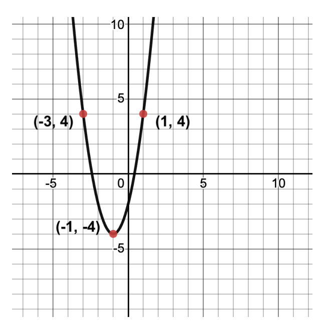
$h(x) = -\ (x - 2)^2 + 3$ 3 C



$j(x) = 0.25(x - 5)^2 - 4$ 4 D



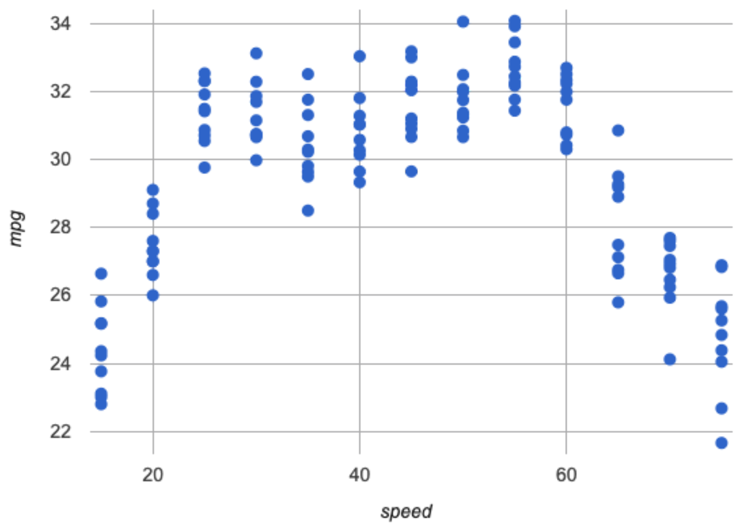
$k(x) = \frac{1}{4}(x - 4)^2 + 1)$ 5 E



Build a Model from Samples

*For this page, you'll need to open the [Fuel Efficiency Starter File] on your computer. If you haven't already, select **Save a Copy** from the "File" menu to make a copy of the file that's just for you. **Read the comments at the top of the file**, which describe what each column in the dataset means.*

The **standard form** of a quadratic equation is $y = Ax^2 + Bx + C$



1) Choose a point from the **left-most column** of dots, and fill in the **standard form** equation below:

______ = A(______)^2 + B(______) + C
y (mpg) x (speed) x (speed)

2) Choose a point from the **center-most column** of dots, and fill in the **standard form** equation below:

______ = A(______)^2 + B(______) + C
y (mpg) x (speed) x (speed)

3) Choose a point from the **right-most column** of dots, and fill in the **standard form** equation below:

______ = A(______)^2 + B(______) + C
y (mpg) x (speed) x (speed)

4) In the space below - or on another sheet of paper - solve this series of equations for A, B, and C:

5) Write your finished model in function and Pyret notation below, then define the function `f` in Pyret and try it out using `fit-model`!

Function Notation	Pyret Notation
$f(x) = A($______$)^2$ + B(______) + C x (speed) x (speed)	`fun f(x): ((________ * num-sqr) + (________ * x)) + ________ end`

Looking up Rows and Columns

We can define names for values in Pyret, the same way we do in math:

```
name = "Shanti"
age = 16
logo = star(50, "solid", "red")
```

When **looking up a data Row** from a Table, programmers use the `row-n` function. This function takes a Table and a Number as its inputs. The numbers tell the computer which Row we want from the Table. *Note: Rows are numbered starting at zero!*
For example:

```
sasha   = row-n(animals-table, 0)  # define Sasha to be the first row
mittens = row-n(animals-table, 2)  # define Mittens to be the third row
```

When we define these rows, it's more useful to name them based on their *properties*, rather than their identifiers:

```
cat-row = row-n(animals-table,  0) # Sasha is a cat
dog-row = row-n(animals-table, 10) # Toggle is a dog
```

When **looking up a column** from a Row, programmers use square brackets and the name of the column they want.
For example:

```
# these two lines do the same thing! We can use the defined name to simplify our code
row-n(animals-table, 0)["age"] # look up Sasha's age (in row 0)
cat-row["species"]             # look up Sasha's age (using the defined name)
dog-row["age"]                 # look up Toggle's age (using the defined name)
```

Lookup Questions

The table below represents four pets at an animal shelter:

`pets-table`

name	sex	age	pounds
"Toggle"	"female"	3	48
"Fritz"	"male"	4	92
"Nori"	"female"	6	35.3
"Maple"	"female"	3	51.6

1) *Match* each Lookup Question (left) to the code that will give the answer (right).

"How much does Maple weigh?"	1	A	`row-n(pets-table, 3)`
"Which is the last row in the table?	2	B	`row-n(pets-table, 2)["name"]`
"What is Fritz's sex?"	3	C	`row-n(pets-table, 1)["sex"]`
"What's the third animal's name?"	4	D	`row-n(pets-table, 3)["age"]`
"How much does Nori weigh?"	5	E	`row-n(pets-table, 3)["pounds"]`
"How old is Maple?"	6	F	`row-n(pets-table, 0)`
"What is Toggle's sex?"	7	G	`row-n(pets-table, 2)["pounds"]`
"What is the first row in the table?"	8	H	`row-n(pets-table, 0)["sex"]`

2) For each value on the left, write the Pyret expression that will produce that value on the right. The first one has been completed for you.

a.	"Maple"	row-n(pets-table, 3)["name"]
b.	"male"	
c.	4	
d.	48	
e.	"Nori"	

More Practice with Lookups

Consider `shapes-table` below, and the four value definitions that follow.

name	corners	is-round
"triangle"	3	false
"square"	4	false
"rectangle"	4	false
"circle"	0	true

```
shapeA = row-n(shapes-table, 0)
shapeB = row-n(shapes-table, 1)
shapeC = row-n(shapes-table, 2)
shapeD = row-n(shapes-table, 3)
```

1) *Match* each Pyret expression (left) to the description of what it evaluates to (right).

`shapeD`	1	A	Evaluates to 4
`shapeA`	2	B	Evaluates to the last row in the table
`shapeB["corners"]`	3	C	Evaluates to `"square"`
`shapeC["is-round"]`	4	D	Evaluates to `true`
`shapeB["name"]`	5	E	Evaluates to `false`
`shapeA["corners"]`	6	F	Evaluates to 3
`shapeD["name"] == "circle"`	7	G	Evaluates to the first row in the table

2) For each value on the left, write the Pyret expression that will produce that value on the right. The first one has been completed for you.

a.	"rectangle"	shapeC["name"]
b.	"square"	
c.	4	
d.	0	
e.	true	

Defining Rows

Remember: rows start at index zero!
We've already given you two row definitions, `cat-row` and `dog-row`:

```
cat-row = row-n(animals-table,  0)  # Sasha is a cat
dog-row = row-n(animals-table, 10)  # Toggle is a dog
```

1) Use the [Animals Table] to identify the index of a row containing...

a lizard	________________
a rabbit	________________
a fixed animal	________________
a male animal	________________
a female animal	________________
a hermaphroditic animal	________________
an unfixed animal	________________
a young animal (<2 years)	________________
an old animal (>10 years)	________________

2) What code would you write to define `lizard-row`?

____

3) What code would you write to define `rabbit-row`?

____

4) What code would you write to define `fixed-row`?

____

5) What code would you write to define `male-row`?

____

6) What code would you write to define `female-row`?

____

7) What code would you write to define `hermaphrodite-row`?

____

8) What code would you write to define `young-row`?

____

9) What code would you write to define `old-row`?

____

Add this code to your Animals Starter File! You'll want these rows for later!

Exploring the Covid Dataset

*For this page, you'll need to have the [Covid Spread Starter File]() open on your computer. If you haven't already, select **Save a Copy** from the "File" menu to make a copy of the file that's just for you.*

1) Take a look at the Definitions Area and find the "Notes on Columns". What is the start date for the data in this table? ______________

2) Click "Run", and evaluate `covid-table` in the Interactions Area. What do you notice? ______________

____

3) Evaluate `MA1` in the Interactions Area. What does it return? ______________

4) Evaluate `CT1`. What information do you learn? ______________

5) Evaluate `NH1`. Why is it "unbound" and how could we make it work? ______________

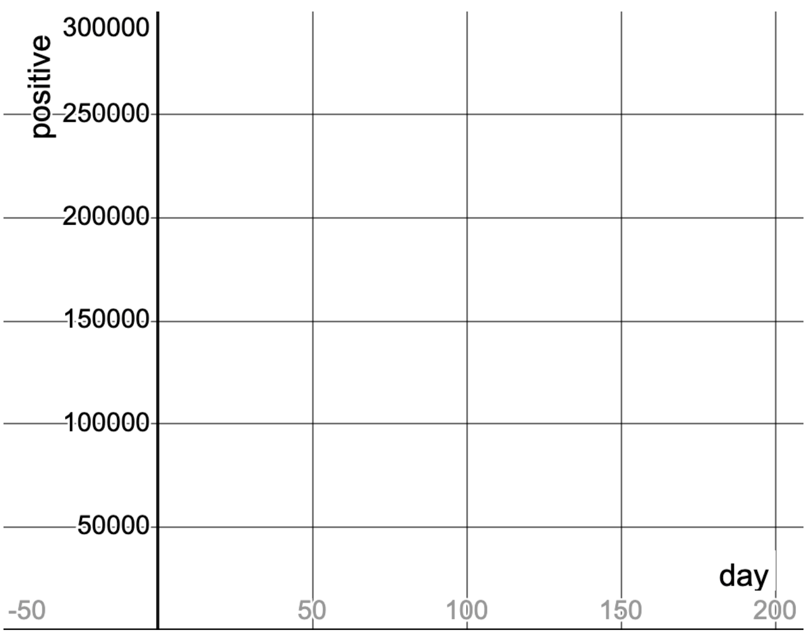
6) Define three new Rows called `NH1`, `RI1` and `VT1`, for New Hampshire, Rhode Island and Vermont. Click "Run" and test them out.

   a. How many people in **Vermont** had tested positive by June 10th, 2020? ______________

   b. How many people in **New Hampshire** tested positive by June 10th, 2020? ______________

   c. How many people in **Rhode Island** tested positive by June 10th, 2020? ______________

7) In Pyret, make a scatter plot showing the relationship between `day` and `positive`, using `state` as your labels, then sketch the resulting scatter plot below.



8) In which state did the number of cases grow *fastest*?

____

9) In which state did the number of cases grow *slowest*?

____

10) Are these strong or weak relationship(s)?

____

11) What do you **Notice?** ______________

____

____

12) What do you **Wonder?** ______________

____

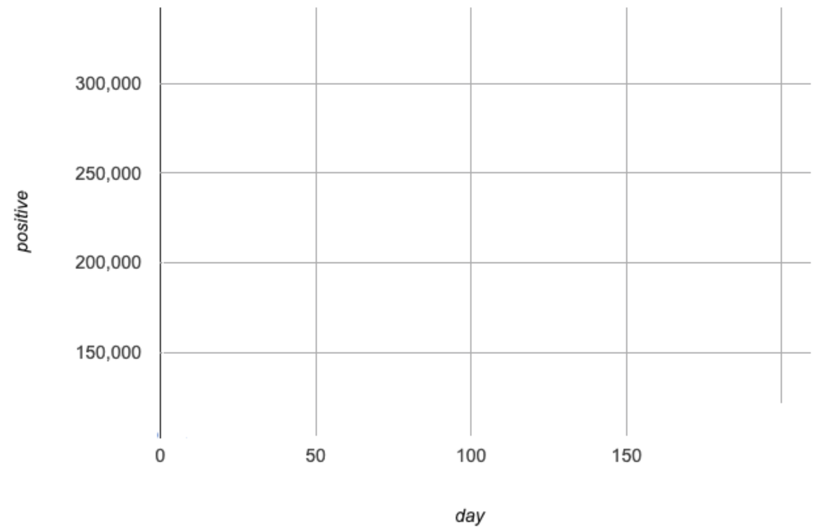
____

Linear Models for MA-table

*For this page, you'll need to have the [Covid Spread Starter File]() open on your computer. If you haven't already, select **Save a Copy** from the "File" menu to make a copy of the file that's just for you.*

This starter file defines a table *just for MA data*, called `MA-table` **:** `MA-table = filter(covid-table, is-MA)`

1) Make a scatter plot from `MA-table` showing the relationship between `day` and `positive`, using `state` as the labels. Sketch the plot on the right.



As we've seen, it's easy to fit a linear model to any dataset in Pyret, so let's start by testing how well a linear function could model this data.

2) Use `lr-plot` to obtain the best-possible **linear model** for the MA Covid dataset, and write it below:

y = ______________________________ $S =$ ______________________________

Note: Pyret uses `e` *for scientific notation. For example:* $2.46e3 = 2.46 \times 10^3 = 2460$

3) The optimized linear model for this dataset predicts an ______ (increase / decrease) of about ______ (slope) ______ (y-variable) per ______ (x-variable).

The error in the model is described by an ***S-value*** of about ______ (S) ______ (units), which is a ______ (poor, ok, good) fit considering that ______ (y-variable) in this dataset range from ______ (lowest y-value) to ______ (highest y-value).

4) Change the definition of the `linear` function in the [Covid Spread Starter File]() to match the model produced by `lr-plot` and "Save".

5) Do you think a linear function is a good model for this dataset? Why or why not? ______________________________

____

____

____

★ What do you think the code that defines `MA-table` is actually doing? ______________________________

____

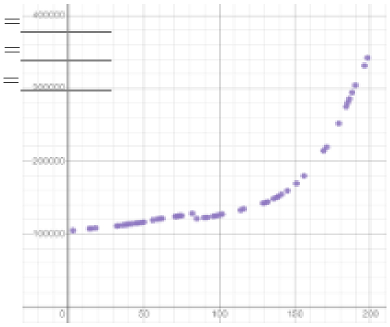
Quadratic Models for MA-table

Fitting the Model Visually $f(x) = a(x - h)^2 + k$

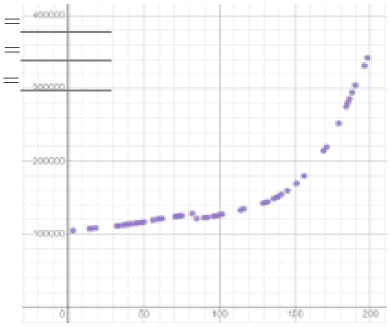
*For this section, you'll need to have **Slide 1: Quadratic Model for MA** of **Modeling Covid Spread (Desmos)** open on your computer.*

1) Try changing the values of a, h and k to find three promising quadratic models, graphing each one and labeling your values in the grids below.

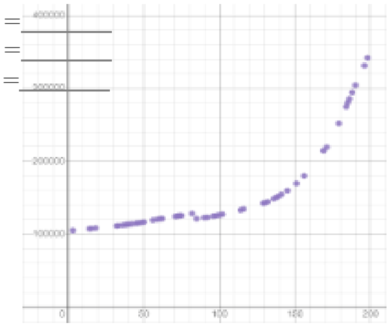
$a =$ ______
$h =$ ______
$k =$ ______



$a =$ ______
$h =$ ______
$k =$ ______



$a =$ ______
$h =$ ______
$k =$ ______



2) Do your quadratic models open up or down? __________. What does that tell us about a? ______________________________.

3) Describe one of your models: Where is the vertex? (________ , ________) What is the horizontal shift? ________ The vertical shift? ________
x y h k

4) Which quadratic form would be the easiest to fit to this data? standard □ factored □ vertex□

Fitting the Model Programmatically $f(x) = a(x - h)^2 + k$

For this section, open your copy of the [Covid Spread Starter File].

5) In the space below, define `quadratic1` to be the first model you fit in Desmos.

```
fun quadratic1(x): ( ________________ * (num-sqr( x - ________________ ))) + ________________ end
```
a h k

6) Return to [Covid Spread Starter File] and update the definitions for `quadratic1`, `quadratic2`, and `quadratic3`.
Then click "Run" to load your updated definition.

7) Use `fit-model` to determine the S-value of each model using the `MA-table`.
Hint: If you forgot the contract for `fit-model`, look it up in the [contracts pages]!

S for `quadratic1`: ________________ S for `quadratic2`: ________________ S for `quadratic3`: ________________

What does this model actually mean?

After experimenting, the best quadratic model I came up with for this dataset shows that __________ are correlated to __________.
x-variable y-variable

The vertex of the parabola drawn by this model is a ________________ at about ________________, which predicts that ______________
minima or maxima? (x, y)

____.

The error in the model is described by an ***S-value*** of about ________________________, which is a ________________________
S units bad, ok, good

fit considering that ______________ in this dataset range from ______________ to ______________.
y-variable lowest y-value highest y-value

Are Quadratic Models a Good Fit for This Data?

8) Would you feel good about making predictions based on these models? Why or why not? ______________________________

____

____

What Kind of Model? (Tables)

Decide whether each table is best described by a linear, quadratic, or exponential model. **If the model is exponential:** What is the growth factor? Doubling (factor of 2)? Tripling (factor of 3)? Factor of 5? 10?
HINT: Can you draw the arrows to calculate the first difference? The second? *What does it mean if neither one is constant?*

x	y
1	5
2	10
3	15
4	20
5	25
6	30
7	35

1) Linear Quadratic Exponential ______ factor

x	y
0	10
1	100
2	1000
3	10000
4	100000
5	1000000
6	10000000

2) Linear Quadratic Exponential ______ factor

x	y
70	-169
71	-126
72	-81
73	-34
74	15
75	66
76	119

3) Linear Quadratic Exponential ______ factor

x	y
-3	36
-2	16
-1	4
0	0
1	4
2	16
3	36

4) Linear Quadratic Exponential ______ factor

x	y
0	3
1	6
2	12
3	24
4	48
5	96
6	192

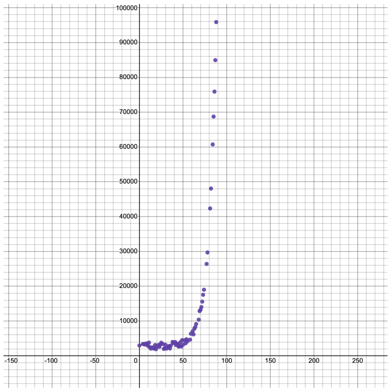
5) Linear Quadratic Exponential ______ factor

x	y
-5	466656
-4	7776
-3	1296
-2	216
-1	36
0	6
1	1

★ Linear Quadratic Exponential ______ factor

What Kind of Model? (Graphs & Plots)

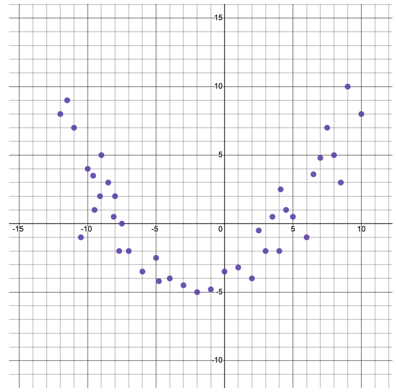
Are these scatter plots best be described by linear, quadratic, or exponential models? **If it's exponential, draw the asymptote!**



1) Linear Quadratic Exponential

How did you know? ______________________________

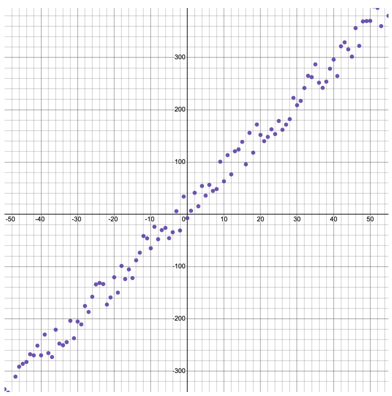
____



2) Linear Quadratic Exponential

How did you know? ______________________________

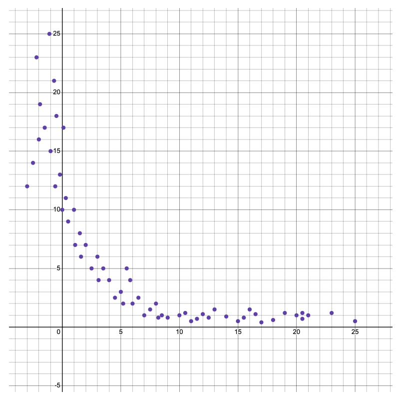
____



3) Linear Quadratic Exponential

How did you know? ______________________________

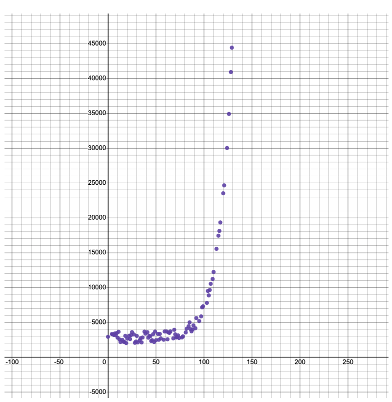
____



4) Linear Quadratic Exponential

How did you know? ______________________________

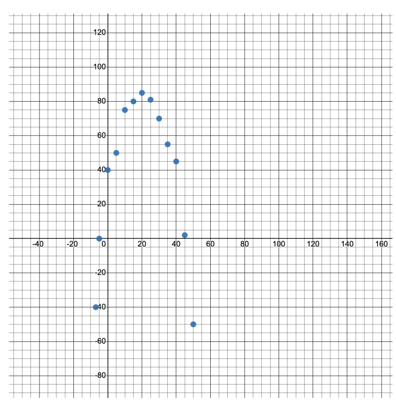
____



5) Linear Quadratic Exponential

How did you know? ______________________________

____



6) Linear Quadratic Exponential

How did you know? ______________________________

____

Graphing Exponential Models: *f(x) = ab^x + k*

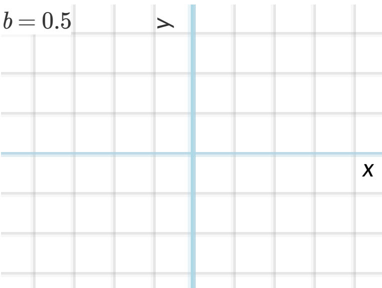
*For this page, you'll need to have **Slide 3: Exploring Exponential Models** of **Modeling Covid Spread (Desmos)** open on your computer.*
The curve you'll see is the graph of $h(x) = 2^x$. Another curve $f(x)$ is hiding behind it with identical coefficients: $k = 0$, $b = 2$ and $a = 1$.

Base b

1) Make sure $k = 0$ and $a = 1$. Experiment with b. **For what values of b is the function *undefined***, with the line disappearing? ____________

2) Keeping $a = 1$ and $k = 0$, change b to 0.5, 1, and 2, graphing each curve below. For each curve, **label the coordinates at x=1, 2, and 3.**

$b = 0.5$



$b = 1$

$b = 2$

3) What does b tell us about an exponential function, when $b > 1$? ______________________________

____

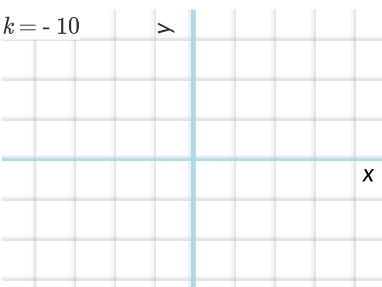
4) What does b tell us about an exponential function, when $0 < b < 1$? ______________________________

____

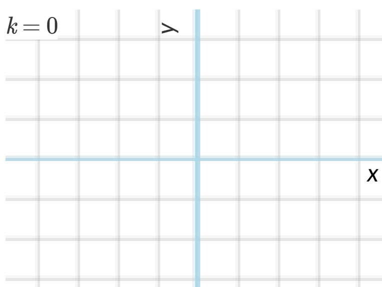
Vertical Shift…and Horizontal Asymptote k

5) Keeping $a = 1$ and $b = 2$, try changing the value of k to -10, 0, and 10, graphing each curve in the squares below. For each curve, find and label the y-value where the curve is "most horizontal", then **draw a horizontal line at that y-value**.

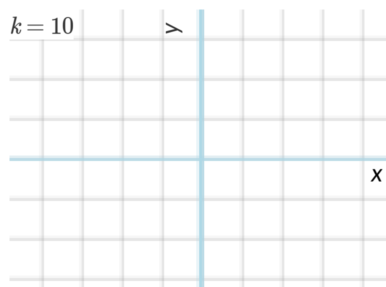
$k = -10$



$k = 0$



$k = 10$



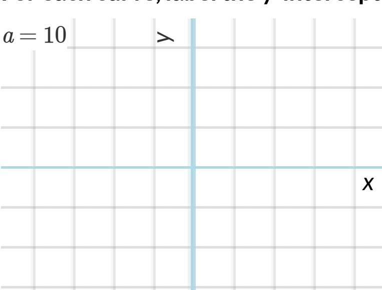
6) What does k tell us about an exponential function? ______________________________

____

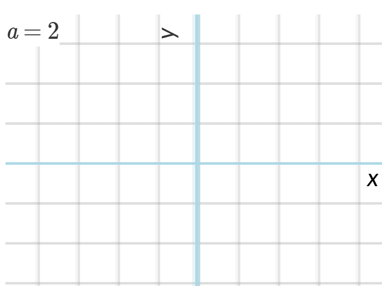
Initial Value a

7) Set $k = 0$ and $b = 2$. Change the value of a to 10, 2, and -5, graphing each curve in the squares below.
For each curve, label the y-intercept (x=0).

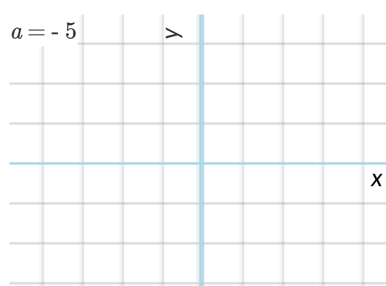
$a = 10$



$a = 2$



$a = -5$



8) What does a tell us about an exponential function? ______________________________

What Kind of Model? (Descriptions)

Decide whether each situation is best described by a linear, quadratic, or exponential function.
If the function is exponential: What is the growth factor. Is it doubling (factor of 2)? Tripling (factor of 3)? Factor of 5? 10?

Car Values

A particular kind of car sells for $32,000, and its resale value drops by 12.5% each year.

1) Is the function increasing or decreasing? ____________________

2) When the car is brand-new (x=0), how much is it worth? ________________

3) How much is it worth after…

(1 year) x=1	(2 years) x=2	x=3	x =4

4) What is the **form** of this function? linear □ quadratic □ exponential□

5) **If it's exponential**,

Fill in the coefficients to write a function that shows the value of the car after a given number of years:

$f(x) = \underset{\text{initial value } a}{\underline{\qquad\qquad}}\ \underset{\text{growth factor } b}{\underline{\qquad\qquad}}^{x} + \underset{\text{horizontal asymptote } k}{\underline{\qquad\qquad}}$

Is it exponential *growth* ? □ or *decay* ? □

Lemonade Stand

Sally is selling lemonade, for $1.25 a glass in hopes of finally be able to get the power drill she's been wanting. She starts with $20 cash.

6) Is the function increasing or decreasing? ____________________

7) When Sally starts the day (x=0), how many dollars does she have? ________________

8) How many dollars will she have after…

(first sale) x=1	(second sale) x=2	x =3	x = 4

9) What is the **form** of this function? □ linear □quadratic □ exponential

10) **If it's exponential**,

Fill in the coefficients to write a function that shows how much Sally has saved after a given number of sales:

$f(x) = \underset{\text{initial value } a}{\underline{\qquad\qquad}}\ \underset{\text{growth factor } b}{\underline{\qquad\qquad}}^{x} + \underset{\text{horizontal asymptote } k}{\underline{\qquad\qquad}}$

Is it exponential *growth* ? □ or *decay* ? □

What Kind of Model? (Definitions)

Decide whether each representation describes a linear, quadratic, or exponential function.
If the function is exponential: Identify the growth factor and the initial value.

$$f(x) = 6x^2 - 5$$

1) Linear Quadratic Exponential

How did you know? ______________________________

____

If it's exponential, what's the ________ (growth factor) ________ (initial value)?

$$\text{miles}(\text{hours}) = \frac{22 \times \text{hours} + 14}{12 - 9}$$

2) Linear Quadratic Exponential

How did you know? ______________________________

____

If it's exponential, what's the ________ (growth factor) ________ (initial value)?

$$\text{cost}(w) = 5(1.2^w) + 16$$

3) Linear Quadratic Exponential

How did you know? ______________________________

____

If it's exponential, what's the ________ (growth factor) ________ (initial value)?

$$t(g) = 42 - 2g^2$$

4) Linear Quadratic Exponential

How did you know? ______________________________

____

If it's exponential, what's the ________ (growth factor) ________ (initial value)?

$$\text{price}(d) = d^2 + 6d$$

5) Linear Quadratic Exponential

How did you know? ______________________________

____

If it's exponential, what's the ________ (growth factor) ________ (initial value)?

$$j(x) = \frac{1}{2}^x + 22$$

6) Linear Quadratic Exponential

How did you know? ______________________________

____

If it's exponential, what's the ________ (growth factor) ________ (initial value)?

$$f(a) = 20000 - 4.1^a$$

7) Linear Quadratic Exponential

How did you know? ______________________________

____

If it's exponential, what's the ________ (growth factor) ________ (initial value)?

$$g(x) = 8(3^{-4x})$$

8) Linear Quadratic Exponential

How did you know? ______________________________

____

If it's exponential, what's the ________ (growth factor) ________ (initial value)?

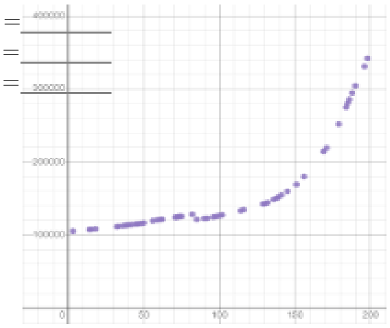
Exponential Models: *f(x) = abˣ + k*

Fitting the Model Visually - MA

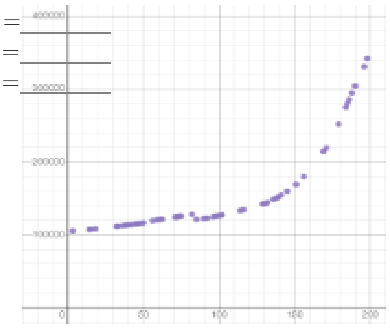
For this section, you'll need to have ***Slide 4: Exponential Model for MA*** *of* ***Modeling Covid Spread (Desmos)*** *open on your computer.*

1) Try changing the value of k, then a, then b to find three promising exponential models, graphing each one and labeling your values on the grids below.

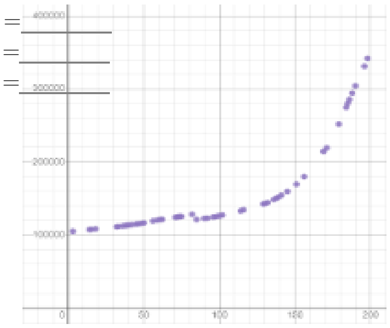
$a =$ ______
$b =$ ______
$k =$ ______



$a =$ ______
$b =$ ______
$k =$ ______



$a =$ ______
$b =$ ______
$k =$ ______



Fitting the Model Programmatically - MA

For this section, open your copy of the Covid Spread Starter File.

2) In the space below, define `exponential` for one of the models you fit in Desmos.

```
fun exponential(x): ( ____________ * num-expt( ______________ , (~1 * x))) + ______________ end
                           a                          b                                k
```

Two Notes on this function definition:

- `num-expt` *is the function that we use for exponents. It takes in 2 numbers: the base and the power, in this case b and x.*
- `(~1 * x)` *at first it may appear that x is being multiplied by negative 1, but it is actually being multiplied by* ***~1*** *(literally the value "roughly 1"). This tells Pyret to round off the calculation, prioritizing* ***speed*** *over* ***precision*** *to get a result that is "roughly accurate". We've added this to the function definition so that you won't have to wait for several minutes for Pyret to run* `fit-model` *to get an answer for question 4.*

3) Update the definition for `exponential` in the Definitions Area and click "Run" to reload it.
Then use `fit-model` to determine how closely `exponential` fits the `MA-table` and fill in the blanks below to interpret the model.
Hint: If you forgot the contract for `fit-model`*, look it up in the contracts pages!*

According to this exponential model, on June 9, 2020 (day zero) there were about ______ (a) + ______ (k) ______________ (y-units) in MA, for a total of about __________ (a + k). This number grew exponentially, increasing by a factor of __________ (Growth Factor: b) or ____________ (Growth Rate: (b - 1) × 100) % every day.

The error in the model is described by an ***S-value*** of about ________ (S) ________ (units), which is a(n) __________ (bad, ok, good) model considering that ______________ (y-units) in this dataset range from ________ (lowest y-value) to ________ (highest y-value).

4) Estimate how many positive cases there will be after X days by **looking at graph with your eyes**, then use your model to find the answer.

Using your...	Eyes	Model	***Using your...***	Eyes	Model	***Using your...***	Eyes	Model
50 days	______	______	150 days	______	______	250 days	______	______
350 days	______	______	450 days	______	______	550 days	______	______

★ Rewrite the model to make Pyret do these calculations with extreme precision. (Remove the part where it multiplies by `~1`.)
WARNING: Be sure to save your work first, as there's a good chance this will lock up your browser and require force-quitting!

What changed? ____

Data scientists perform calculations to do things like send satellites to far-away planets, or analyze large populations of a billion or more.
You know that the pros of using `~1` involve speed. **What are the potential downsides of using `~1` to speed up a calculation?**

____

____

Modeling Other States

*For this page, you'll need to have the Covid Spread Starter File open on your computer. If you haven't already, select **Save a Copy** from the "File" menu to make a copy of the file that's just for you.*

1) Find the function called `is-MA` in the Definitions Area under "Define some helper functions" and read the comments carefully!

a. What is the Domain of `is-MA`? ______________________ What is its Range? ______________

b. What do you *think* `is-MA(MA1)` will evalute to? __________. `is-MA(CT1)`? __________. `is-MA(ME1)`? ________

Try typing each of the `is-MA` expressions into the Interactions Area on the right and confirm you were correct.

2) Find `MA-table` in the Definitions Area under "Define some grouped and/or random samples". What is that code doing? __________

____

____

3) **Define a new function** `is-VT` and **create a new grouped sample** called `VT-table`.

Hint: You can use the code for `is-MA` and `MA-table` as a model.

Modeling VT

*For this section, in addition to Pyret, you will need to have **Slide 5: Exponential Model for VT** of **Modeling Covid Spread (Desmos)** open on your computer.*

4) Use `lr-plot` to obtain the best-possible linear model for the relationship between `day` and `positive` in the `VT-table`, then fill in the blanks below:

The optimized linear model for this dataset predicts an ____________ (increase / decrease) of about ______ (slope) ____________ (y-variable) per ____________ (x-variable).

The error in the model is described by an ***S-value*** of about ______ (S) ______ (units), which is ____________ (insignificant, moderate, significant, extreme)

considering that ____________ (y-variable) in this dataset range from ____________ (lowest y-value) to ____________ (highest y-value).

5) Use **Slide 5: Exponential Model for VT** of **Modeling Covid Spread (Desmos)** to come up with the best exponential model you can for the Vermont dataset, and write it below:

____

6) Add a definition for `exponential-VT` to the Definitions area of Covid Spread Starter File using the model you just found.

- Click "Run" to load your definition.
- Then fit the model using `VT-table`

According to this exponential model, on June 9, 2020 (day zero) there were about ______ (a) + ______ (k) ____________ (y-units) in VT, for a total

of about __________ (a + k). This number grew exponentially, increasing by a factor of __________ (Growth Factor: b) or ____________ (Growth Rate: (b - 1) × 100) % every

day. The error in the model is described by an ***S-value*** of about ______ (S) ______ (units), which is

____________ (insignificant, moderate, significant, extreme) considering that __________ (y-units) in this dataset range from __________ (lowest y-value) to __________ (highest y-value).

7) Are exponential models a good fit for this data? Why or why not? ______________________

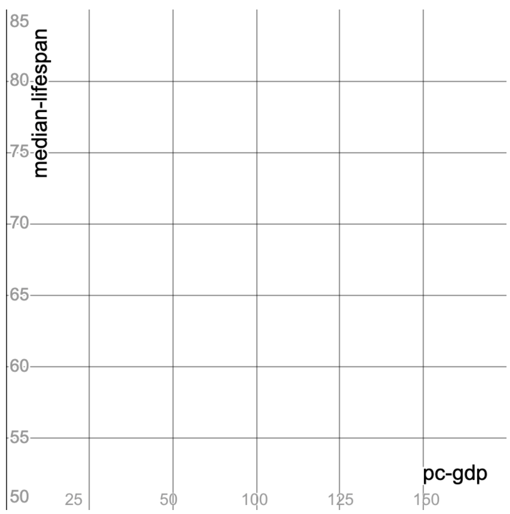
____

Exploring the Countries Dataset

*For this section, you'll need the [Countries of the World Starter File] open on your computer. If you haven't already, select **Save a Copy** from the "File" menu to make a copy of the file that's just for you. The columns in this dataset are described below:*

- **country** - name of the country
- **gdp** - total Gross Domestic Product of the country. GDP is often used to measure the economic health of a country.
- **population** - number of people in the country
- **pc-gdp** - the average GDP *per-person, in thousands* of $US
- **has-univ-healthcare** - indicates if the country has universal healthcare
- **median-lifespan** - the median life expectancy of people in the country

1) Make a scatter plot showing the relationship between `pc-gdp` and `median-lifespan`, and sketch its plot below.



2) What do you **Notice?** ______________________________

____

____

3) What do you **Wonder?** ______________________________

____

____

4) Are there any countries that stand out? Why or why not? ________

____

____

5) Suppose a wealthy country is suffering heavy casualties in a war. Draw a star on the plot, showing where you might expect it to be.

6) Do you think you see a relationship? If so, describe it. Is it linear or nonlinear? Strong or weak?

____

____

____

Fitting Models for the Countries Dataset

*For this page you will be working with both the [Countries of the World Starter File]() and the **Desmos** file **Fitting Wealth-v-Health and Exploring Logarithmic Models**.*

Find linear, quadratic and exponential models for the relationship between `pc-gdp` and `median-lifespan`. As you find each model:

- update the corresponding definition in the Countries of the World Starter File
- click "Run" to load your new definition
- use `fit-model` to calculate the ***S-value*** *Hint: If you forgot the contract for `fit-model` (to calculate S), look it up in the contracts pages!*

1) Find the optimized **linear model** for this data using `lr-plot`.

$linear(x) =$ ____________ (slope (m)) x + ____________ (y-intercept / vertical shift) ____________ (S-value)

The optimized linear model for this dataset predicts that a ________ (x-units) ____________ (increase / decrease) in per-capita gdp (x-variable) will increase

____________ (y-variable) by ____________ (y-units). The error in the model is described by an S - $value$ of about ________ (S) ________ (y-units),

which is ____________ (insignificant / reasonable / significant / extreme) considering ________ (y-units) in this dataset range from ________ (lowest y-value) to ________ (highest y-value).

2) Find the best **quadratic model** you can, using the second slide *(Wealth-v-Health Quadratic)* in the Desmos activity.

$quadratic(x) =$ ____________ (quadratic coefficient (a)) $(x -$ ____________ (horizontal shift (h)) $)^2 +$ ____________ (vertical shift (k)) ____________ (S-value)

The vertex of the parabola drawn by my model is a ____________ (minima or maxima?) at about (________ (x, y)).

- *Before* this point, as ____________ (x-variable) increases, ____________ (y-variable) ____________ (increases or decreases?).
- *After* this point, as ____________ (x-variable) increases, ____________ (y-variable) ____________ (increases or decreases?).

The error in the model is described by an S - $value$ of about ________ (S) ________ (y-units), which is ____________ (insignificant / reasonable / significant / extreme)

considering ____________ (y-units) in this dataset range from ________ (lowest y-value) to ________ (highest y-value).

3) Find the best **exponential model** you can, using the third slide *(Wealth-v-Health Exponential)* in the Desmos activity.

$exponential(x) =$ ____________ (initial value (a)) (____________ (growth factor (b)) ^{x}) $+$ ____________ (vertical shift (k)) ____________ (S-value)

According to this exponential model, a country with a ____________ (x-variable) of zero ____________ (x-unit) would have a

____________ (y-variable) of ________ (a) + ________ (k) ________ (y-units), for a total of about ________ (a + k). This number grows exponentially,

increasing by a factor of ________ (Growth Factor: b) or ________ (Growth Rate: (b - 1) × 100) % with every ____________ (x-unit) increase in ____________ (x-variable).

The error in the model is described by an S - $value$ of about ________ (S) ________ (y-units), which is ____________ (insignificant / reasonable / significant / extreme)

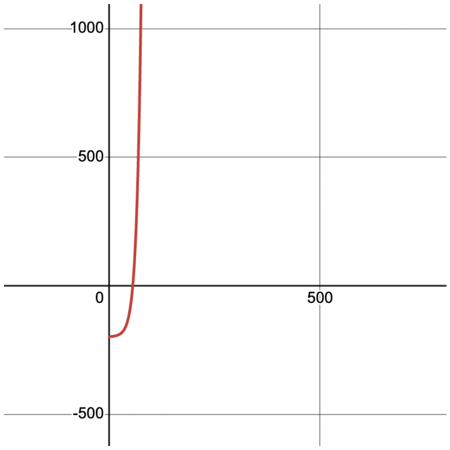
considering ____________ (y-units) in this dataset range from ________ (lowest y-value) to ________ (highest y-value).

4) Are any of these models a good fit for this data? Why or why not?

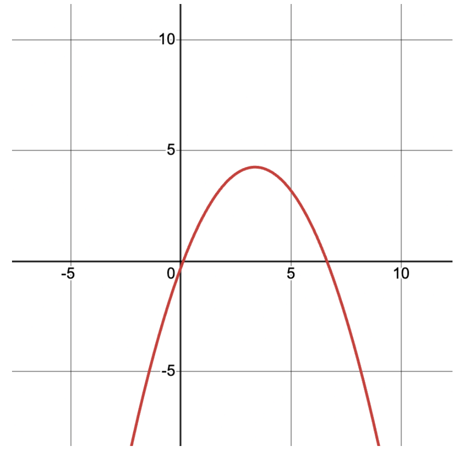
____

What Kind of Model? (Graphs & Plots)

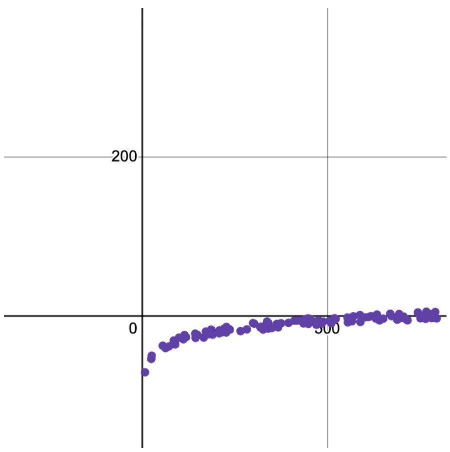
Decide whether each representation is best described by a quadratic, exponential, or logarithmic function. **If you think it's exponential OR logarithmic, draw a diagonal line for $y = x$, and then sketch the reflection of the curve.**



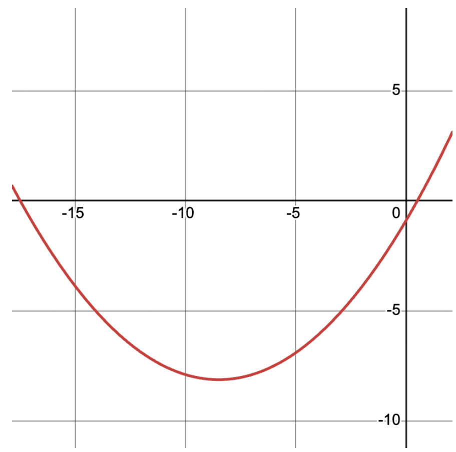
1) Quadratic Exponential Logarithmic



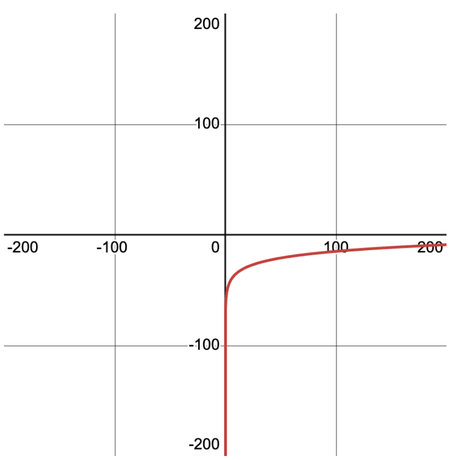
2) Quadratic Exponential Logarithmic



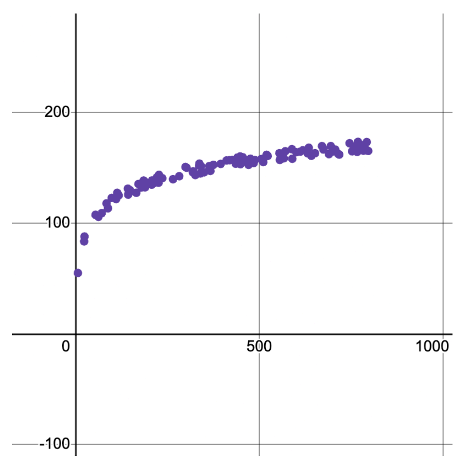
3) Quadratic Exponential Logarithmic



4) Quadratic Exponential Logarithmic



5) Quadratic Exponential Logarithmic



6) Quadratic Exponential Logarithmic

What Kind of Model? (Tables)

Decide whether each representation is best described by a quadratic, exponential, or logarithmic function.
If the function is exponential, find the *base* (also called the *growth factor*): How much does y increase (2x? 10x?) for a single increase in x?
If the function is logarithmic, find the *base*: How much does x need to increase (2x? 10x?) just to get a single increase in y?
HINT: Can you draw the arrows to calculate the first difference? The second? *What does it mean if neither one is constant?*

x	y
1	0
10	1
100	2
1000	3
10000	4
100000	5
1000000	6

1) Quadratic Exponential ____ (base) Logarithmic ____ (base)

x	y
0	1
1	10
2	100
3	1000
4	10000
5	100000
6	1000000

2) Quadratic Exponential ____ (base) Logarithmic ____ (base)

x	y
70	-169
71	-126
72	-81
73	-34
74	15
75	66
76	119

3) Quadratic Exponential ____ (base) Logarithmic ____ (base)

x	y
5	1
10	2
20	3
40	4
80	5
160	6
320	7

4) Quadratic Exponential ____ (base) Logarithmic ____ (base)

x	y
-3	36
-2	16
-1	4
0	0
1	4
2	16
3	36

5) Quadratic Exponential ____ (base) Logarithmic ____ (base)

x	y
1	0
6	1
36	2
216	3
1296	4
7776	5
466656	6

6) Quadratic Exponential ____ (base) Logarithmic ____ (base)

Evaluating Logarithmic Expressions

	Expressions	Translation	Evaluates to:
1	$\log_2(8)$	"The power you raise 2 to get 8"	3
2	$\log_2(1)$	"The power you raise 2 to get 1"	0
3	$\log_5(25)$	"The power you raise ________ to get ________"	
4	$\log_5(1)$	"The power you raise ________ to get ________"	
5	$\log_3(81)$	"The power you raise ________ to get ________"	
6	$\log_3(1)$	"The power you raise ________ to get ________"	
7	$\log_2(16)$		
8	$\log_2(32)$		
9	$\log_{10}(1000)$		
10		"The power you raise 0.1 to get 0.01"	
11		"The power you raise 4 to get 64"	
12		"The power you raise 4 to get 1"	

Graphing Logarithmic Models: *f(x) = a log$_b$ x + k*

*Use this page with **Slide 4: Exploring Logarithmic Functions** of **Fitting Wealth-v-Health and Exploring Logarithmic Models (Desmos)**.*

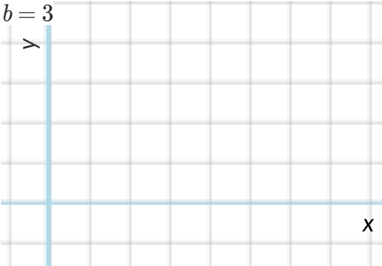
- The **blue curve** is the graph of $h(x) = 1 \log_2 x + 0$. Its constants will remain set at $a = 1$, $b = 2$, and $k = 0$.
- You can modify the **red curve** $g(x)$ (which is hiding behind $h(x)$!) by changing its coefficients: a, b, and k.

Base b

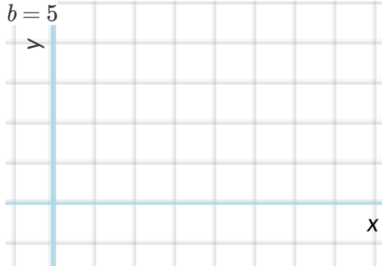
*Keep **k** at 0 and **a** at 1. Change the value of **b** as indicated on each grid below.*

1) Sketch each graph and **label the coordinates where** $x = 1$, $y = 1$, $y = 2$ **and** $y = 3$.

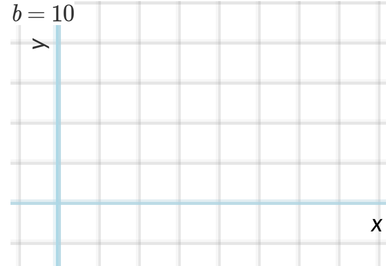
$b = 3$



$b = 5$



$b = 10$



2) How does the value of b impact the shape of a logarithmic function? ____

3) What connections can you draw between the value of b and exponents? ____

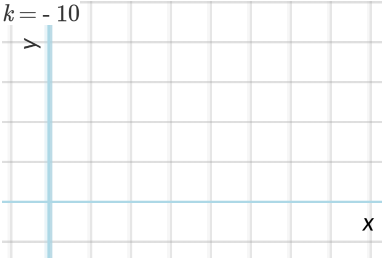
____

Vertical Shift k

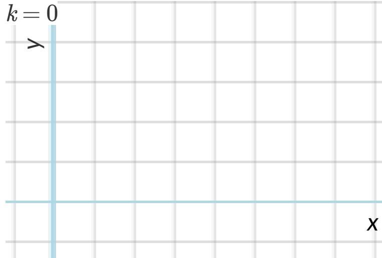
*Set **a** to 1 and **b** to 2. Change the value of **k** as indicated on each grid below.*

4) Sketch each graph and **label the coordinate where** $x = 1$.

$k = -10$



$k = 0$



$k = 10$

5) How does the value of k impact the shape of a logarithmic function? ____

6) Why does $y = k$ when $x = 1$? ____

Logarithmic Coefficient a

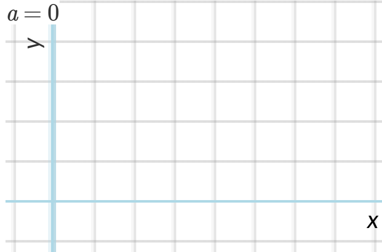
*Set **k** to 0 and **b** to 10, then zoom out out so you can see as far as $x = 1{,}000$.*

Change $h(x)$ to $h(x) = 1 \log_{10}(x) + 0$ so that the blue curve lands on top of the red curve.

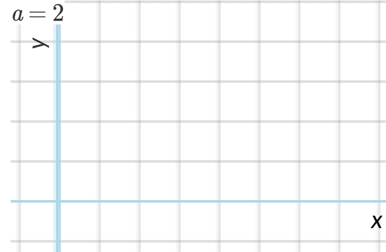
7) In each graph, **label the coordinates where** $x = 10$ **and** $x = 100$ **and** $x = 1000$.

$a = -2$

$a = 0$



$a = 2$



8) What is the value of x when $1\log_2(x) = 4$? __________ What about when $2\log_4(x) = 4$? __________ When $3\log_8(x) = 4$? __________

★ How are a and b related? ____

What Kind of Model? (Descriptions)

Decide whether each situation describes a quadratic, exponential, or logarithmic function. **HINT:** draw a table and plug in some points!

1) Earthquakes release enormous amounts of energy, which we can compare to the energy released by blowing up pounds of dynamite. For example, $\text{richter}(12{,}000) = 4.0$, meaning that the force of blowing up 12,000 pounds of dynamite produces a 4.0 on the Richter scale! $\text{richter}(400{,}000) = 5.0$, $\text{richter}(12{,}540{,}000) = 6.0$, and $\text{richter}(398{,}000{,}000) = 7.0$.

Quadratic Exponential Logarithmic

2) A car accelerates at a constant rate of 5mph/s. After 1 second, $\text{distance}(1) = 2.5\text{miles}$. $\text{distance}(2) = 10$, $\text{distance}(3) = 22.5$, and $\text{distance}(4) = 40$

Quadratic Exponential Logarithmic

3) Moore's law says that the number of transistors in a microprocessor will double roughly every 1.5 years. Starting with 16 transistors, how many years will it take to reach 4,294,967,296 transistors?

Quadratic Exponential Logarithmic

4) The population of a colony of bacteria can double every 20 minutes, as long as there is enough space and food. Starting with 1 bacteria, $f(20) = 2$, $f(40) = 4$, $f(60) = 8$, $f(80) = 16\ldots$

Quadratic Exponential Logarithmic

5) Sequan puts \$100 in a savings account, earning 4% interest. After a year, $\text{savings}(1) = \$104$. $\text{savings}(2) = \$108.16$, $\text{savings}(2) = \$112.49\ldots$

Quadratic Exponential Logarithmic

6) If the *width and length* of a rectangle doubles, how much does the *area* change?

Quadratic Exponential Logarithmic

Changing the Scale

For this page, you'll need to have ***Slide 5: Wealth-v-Health (Logarithmic)*** *of* ***Fitting Wealth-v-Health and Exploring Logarithmic Models (Desmos)*** *and [Countries of the World Starter File](#) open on your computer.*

Fitting a Logarithmic Model $f(x) = a \log_b x + k$

Open the `Data Table` **folder by clicking on the triangle (▶)**

- x_1 is the per-capita income for each country in thousands of $US, and y_1 is the median lifespan.
- Next to y_1 you'll see a dark circle with spots (⁘) inside. If the circle is dark, that means that those points are visible on our graph. Click the circle to "turn off" those dots, then click it again to turn them back on.
- Move the graph by clicking and dragging the background.
- Notice that a magnifying glass (🔍) appears to the bottom left of the table. (You may have to scroll down to see the bottom of the table!) Clicking on the magnifying glass resizes/rescales the graph to fit all the points in the table.

1) Write the numbers you see along the x-axis, from left to right: ______________________________

Continue this pattern - what would the next three numbers be? ______________________________

2) Circle the type of function that describes this pattern: Linear Quadratic Exponential

3) Move the sliders for a and c to create the best-fitting logarithmic model you can find, and write it below.
Note: The Bootstrap Pyret function `log` *always uses* $b = 10$*.*

$logarithmic(x) =$ ______________ (log coefficient (a)) $\log_{10}(x) +$ ______________ (vertical shift (k))

`fun logarithmic(x): ( _______ * log(x)) + _______ end`

4) Modify `logarithmic(x)` in [Countries of the World Starter File](#) to define this model, and fit it using `fit-model`.

The error in the model is described by an S - $value$ of about ________ (S) ________ (units), which is ______________________ (insignificant / reasonable / significant / extreme)

considering ______________________ (y-variable) in this dataset ranges from ____________ (lowest y-value) to ____________ (highest y-value).

Scaling the x-Axis

- Click on the wrench button (🔧) in the top-right corner of the Desmos graph to **Open the "Graph Settings" window.**
- **Expand the "More Options" section** by clicking the triangle (▶).
- **Change the x-axis scale** from `Linear` to `Logarithmic`.
- Adjust the view by zooming and dragging the graph to get all of the points in view on the screen and filling most of it.

5) What is the shape of the point cloud *now*, after changing the scale? Linear Quadratic Exponential

6) Write the numbers you see along the x-axis, from left to right: ______________________________

Continue this pattern - what would the next three numbers be? ______________________________

7) Circle the type of function that describes this pattern: Linear Quadratic Exponential

8) Adjust the sliders for a and c to improve the model. *Toggle back and forth between* `logarithmic` *and* `linear` *x-axis scales as you work.* When you are satisfied with your model, record both forms of the definition below.

$logarithmic2(x) =$ ______________ (log coefficient (b)) $\log_{10}(x) +$ ______________ (vertical shift (k))

`fun logarithmic2(x): ( _______ * log(x)) + _______ end`

9) Modify the definition of `logarithmic2(x)` in Pyret to match this model. Use the `fit-model` function to find its ***S-value***: ____________

10) Why do you think transforming the **x-axis** makes our data look linear? ______________________________

____

Transforming the Data

*For this page, you'll need to have **Slide 6: Wealth-v-Health (Transformed)** of **Fitting Wealth-v-Health and Exploring Logarithmic Models (Desmos)** open on your computer.*

- **Find the `Wealth vs. Health` folder, which is open at the top of the expression list**
- This is the same table we've seen before, and the "points" circle (⁖) shows us that these dots are "on" and visible.
- Underneath the `Wealth vs. Health` folder, you'll see a **function** $g(x)$ and a **list** y_2 defined to be the same as y_1.
- **Open the second folder, called `Log(Wealth) vs. Health`, by clicking on the triangle (▶)**

1) Compare the two tables. (Here is a side by side comparison of how they each begin.)

Wealth vs. Health		Log(Wealth) vs. Health		Compare the 2 tables. What do you notice? What do you wonder?
x_1	y_1	$g(x_1)$	y_2	
1.99051	52.1	0.29896436	52.1	
11.76559	78.6	1.0706137	78.6	
15.19295	77.2	1.1816421	77.2	
6.26897	60.6	0.79719619	60.6	
24.95776	76.9	1.3972056	76.9	
20.5888	77.5	1.313631	77.5	

2) Read the comments in rows 3 to 6 of the Desmos file. Where do the x-values in the second table come from? ______________________

____

3) Why is the second column of both tables the same? ______________________

- **Turn the points for the first table OFF, then turn the points for our new table ON.**
  Our log transformation is so drastic that it looks like all the black datapoints are smashed against the y-axis!
- **Rescale** the graph (⊕) to see the cloud.

4) What is the shape of this point cloud? linear☐ quadratic ☐ exponential ☐

5) Why do you think transforming the **x-values** make our data look linear? ______________________

____

____

6) Through trial and error, move the sliders for m and b to create the best-fitting linear model you can find, and write it below.

$f(x) =$ ____________ $x +$ ____________
slope (m) — y-intercept / vertical shift

Let's compare the coefficients from your models.

Linear (From above)	____________ slope (m)	____________ y-intercept / vertical shift
Logarithmic (From Changing the Scale)	____________ log coefficient (a)	____________ vertical shift (k)

7) How are they similar? ______________________

____

Logarithmic Models

Open your copy of the [Countries of the World Starter File] and click "Run".

Transforming: From Logarithmic *Plots* to Linear Ones

1) Find the definition of `g(r)`. What does this function do? ____________________

____

2) Find the Contract for `build-column` on the [Contracts Page].

What is its **Range**? __________ What is its **Domain**? ____________________

3) At the end of the program, you'll find this code:

```
countries-transformed = build-column(countries-table, "log(pc-gdp)", g)
```

What do you think it does? ____________________

____

4) Click "Run", and evaluate `countries-transformed` in the Interactions Area on the right to test it out!

a. What is different about this Table? *Hint: Find the last column!* ____________________

b. Where did the column on the right come from? ____________________

5) Use this new table to make an `lr-plot` comparing `log(pc-gdp)` and `median-lifespan`, with `country` as the label. Record the regression line and S value below:

$y =$ __________ (slope) $x +$ __________ (vertical shift) S: __________

Inverting: From Linear *Models* to Logarithmic Ones

6) Use the coefficients of the *linear* model you just made to complete the *logarithmic* model below:

$logarithmic3(x) =$ __________ (log coefficient (a)) $log_{10}(x) +$ __________ (vertical shift (k)) `fun logarithmic3(x): ( _______ * log(x)) + _______ end`

7) Let's interpret this model:

A country where the ____________________ (x-axis) *is* _______ (base (b)) *times higher than another is also*

predicted to have a ____________________ (y-axis) *that is* __________ (log coefficient (a)) __________ (y-axis units) *longer.*

8) Add the definition of `logarithmic3` to your starter file, and use it with `fit-model` to calculate the value of S: __________

9) Complete the table below, copying your S values from the previous models:

Linear	Quadratic	Exponential	Logarithmic

10) Compare the two smallest S values using percent change. *How much better* is the logarithmic model? __________

____

Exploring Periodic Data

1) According to the table, the ride goes from ____________ feet to ____________ feet in ________ minutes
lowest point / highest point

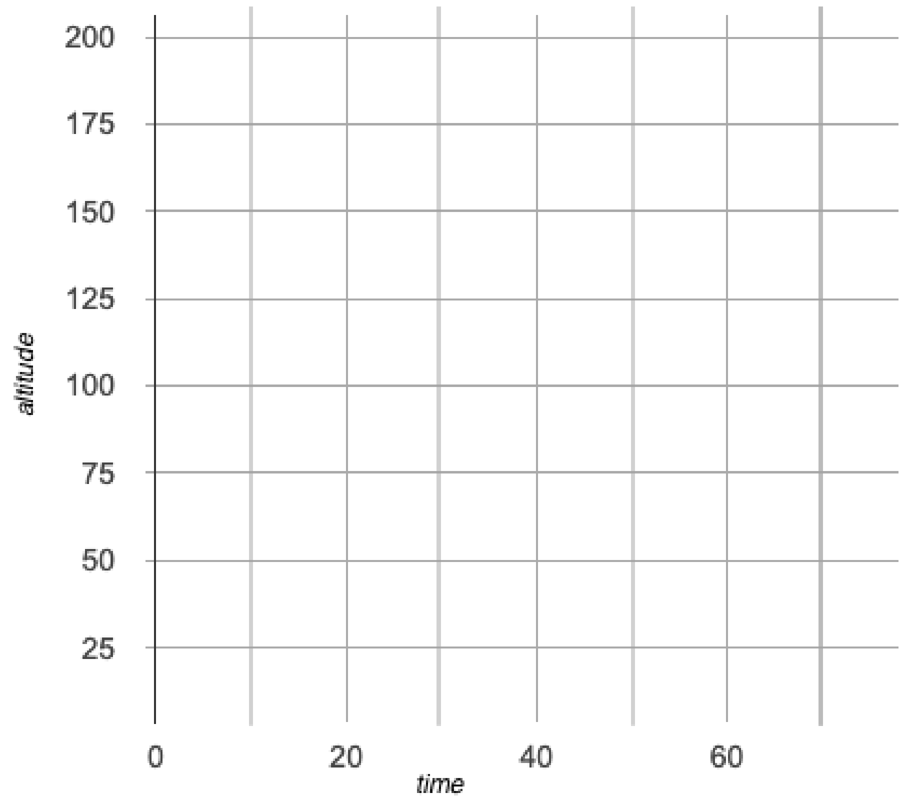
2) It then returns to the lowest point ________ minutes later.

3) Once it gets going, the ride does a **full cycle** from **high-point to high-point** (or low-point to low-point!) in ________ minutes.

4) The altitude **right between** the highest and lowest point is __________ feet. *Draw this as a dashed line on your graph.*

5) Plot each of the points in the table (left) on the coordinate plane (right) to create ***scatter plot***.

`time` (minutes)	`altitude` (feet)
0	5.0
5	55.0
10	154.9
15	205.0
20	155.2
25	55.2
30	5.0
35	54.7
40	154.6
45	205.0
50	155.5
55	55.5
60	5.0



6) What do you **Notice** about the data in the table? ____

____

____

7) What do you **Wonder**? ____

____

8) Working from left to right, *connect the dots* one pair at a time using straight lines. This will create a display known as a **line-graph**.

9) Describe the relationship you see between `time` and `altitude`. (Is it linear, quadratic, exponential, etc.?) ____________________

____

10) What kind of ride do you think your teacher was on, and why? ____

____

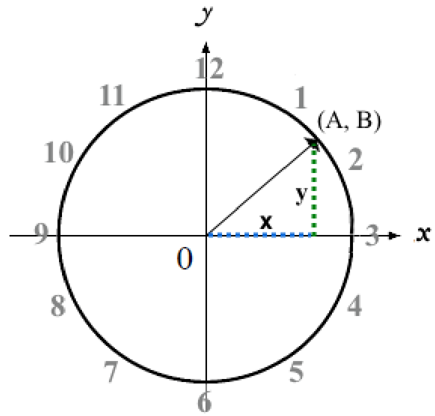
____

Reasoning about Unit Clocks

A *unit clock* (shown below) is centered at the origin $(0, 0)$. As time passes, the point (A, B) rotates around the circle.

1) The radius r of the circle below has a length of 1. What is the length of the **hypotenuse** of the right-triangle formed by A and B? ___________

Time	A
12:00	0
1:30	
3:00	
4:30	
6:00	
7:30	
9:00	
10:30	
12:00	0



Time	B
12:00	1
1:30	
3:00	
4:30	
6:00	
7:30	
9:00	
10:30	
12:00	1

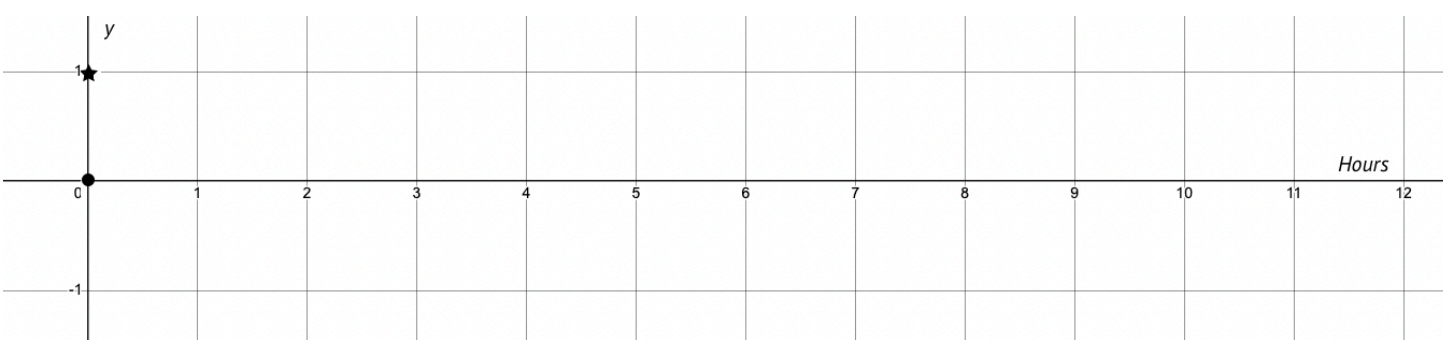
2) The tables above show the values of A and B at 12 o'clock. *Fill in the values of A and B at 3, 6 and 9 o'clock.*

3) In the diagram above, the hand is pointing to (A,B) at **1:30**. At this time, $A = B$. In the space below, (a) draw and label the right triangle, then (2) **fill in the remaining blanks in both tables to show A and B.**

4) Use the values you computed at 1:30 to fill in the rest of the table with values of A and B at 4:30, 7:30, and 10:30.

5) In the graph below, draw a **dot** for the coordinates (*time* , A) in each row of the table. Connect them from left-to-right, to form a curve.

6) In the graph below, draw a **star** for the coordinates (*time* , B) in each row of the table. Connect them from left-to-right, to form a curve.



*Open the Desmos File **Exploring Periodic Functions**. You should be on **Slide 1: Unit Clocks**.*

7) "Turn on" the `x(time)` folder, and compare the graph to your own graph of A. Do they match?

8) Turn off that first folder, and turn on the one for `y(time)`. Compare the graph to your own graph of B. Do they match?

Converting Between Angles

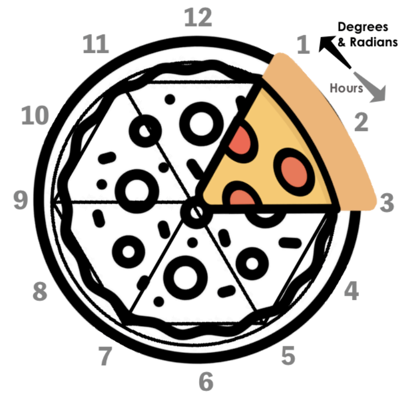
The table below lists different angles within a circle.

1) Fill in the rows for 12, 9, and 6 o'clock, converting between hours, degrees, and radians.

2) Fill in the rows for 1:30, 10:30, 7:30, and 4:30, converting between hours, degrees, and radians.

Remember: degrees and radians both start with zero at "3 o'clock", and increase in the opposite direction of the hours!

We've filled in the rows for 12:00 and 3:00, as well as the length columns.



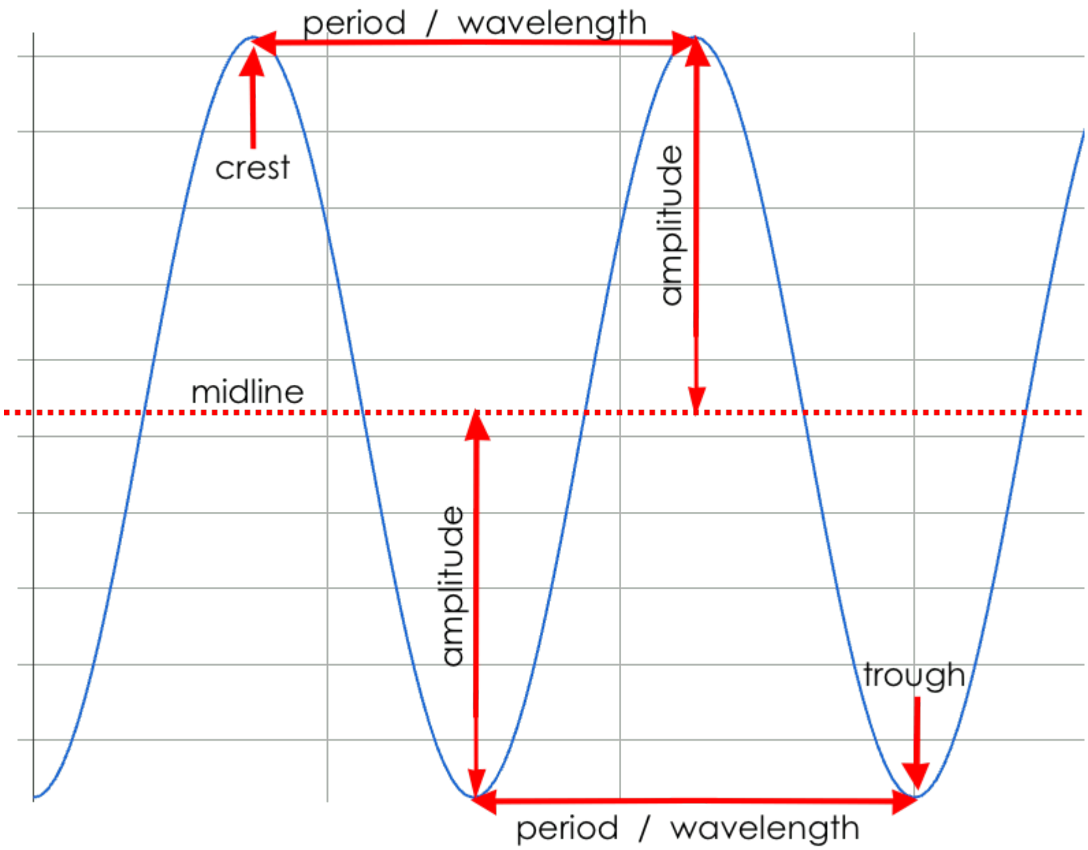
Time	θ Degrees	θ Radians	x	y
3:00	0°	0π	1	0
1:30			$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
12:00	90°	$\frac{2}{4}\pi$	0	1
10:30			$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
9:00			-1	0
7:30			$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$
6:00			0	-1
4:30			$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$
3:00	360°	$\frac{8}{4}pi$	1	0

3) In Pyret, experiment the functions `sin` and `cos`, passing in different **radian values** from the table above.

a. Which function computes $x(\theta)$? __________

b. Which function computes $y(\theta)$? __________

Words for Describing Periodic Functions



Based on what you can learn from the diagram, describe what each of the terms means in your own words.

Peaks - ____

____

Troughs - ____

____

Period - ____

____

Midline - ____

____

Amplitude - ____

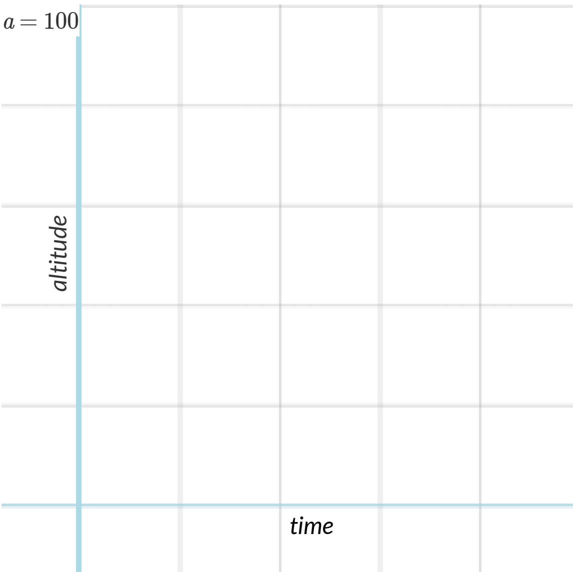
Graphing Periodic Models: Amplitude (a)

The standard form of periodic models is $f(x) = a\ sin(b \cdot (x - h)) + k$. Let's explore the role of ***amplitude*** a in periodic functions! *Open the Desmos File **Exploring Periodic Functions** to Slide 2: **Modeling the Ferris Wheel Dataset (sin)**. You should see four sliders for a, b, h, and k.*

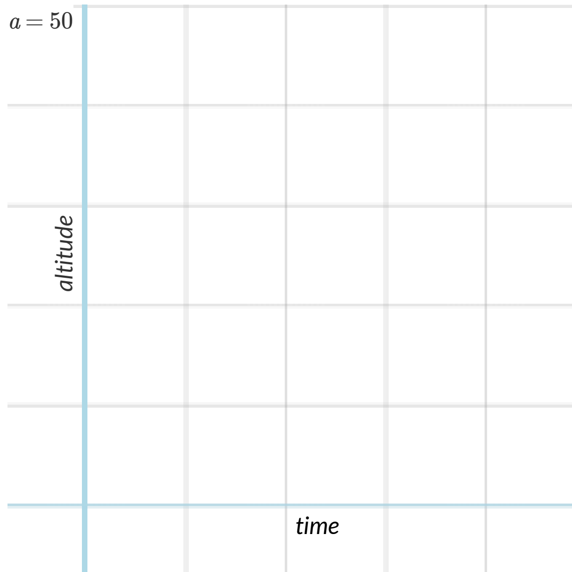
1) Adjust the sliders to fit the data as best you can, and fill in the coefficients: __________ , __________ , __________ and __________
a *b* *h* *k*

2) Change **ONLY** the slider for a, experimenting with values at 100, 50, -50, and 0, graphing each curve below. **For each curve, label the coordinates at time=15, 30, and 45.**

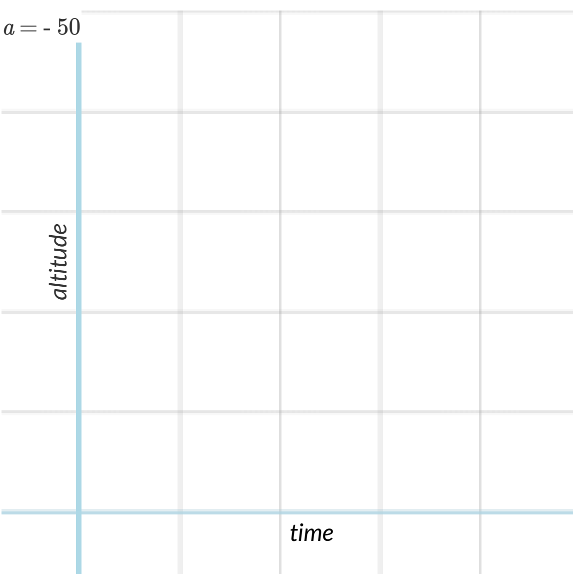
$a = 100$



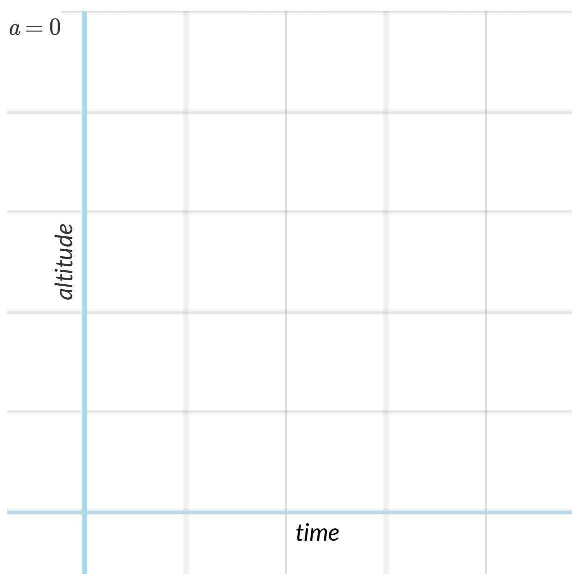
$a = 50$



$a = -50$



$a = 0$



3) What does a tell us about a periodic function? ____________________________________

____

The distance between two adjacent *peaks* or *troughs* is called the *period*: the interval over which the pattern repeats itself.

4) What effect does changing a have on the ***period*** of a periodic function? ____________________________________

Graphing Periodic Models: Frequency (*b*)

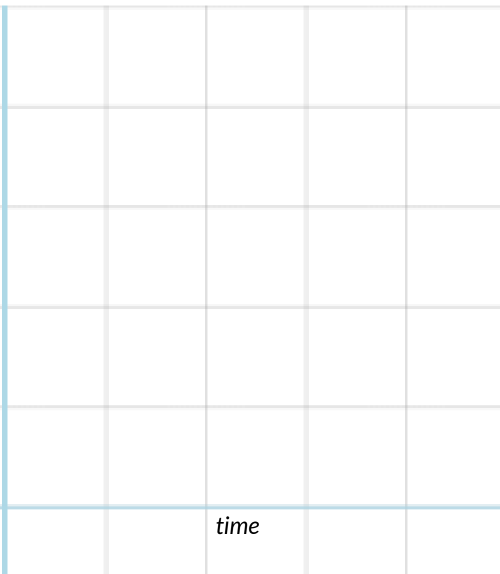
The standard form of a periodic model is $f(x) = a\ sin(b \cdot (x - h)) + k$. On this page, we'll explore the role of ***amplitude*** a in periodic functions. *Open the Desmos File **Exploring Periodic Functions**. You should be on **Slide 2: Modeling the Ferris Wheel Dataset (sin)** and see four sliders for a, b, h, and k.*

1) Adjust the sliders to fit the data as best you can, and fill in the coefficients: ____________ , ____________ , ____________ and ____________
a *b* *h* *k*

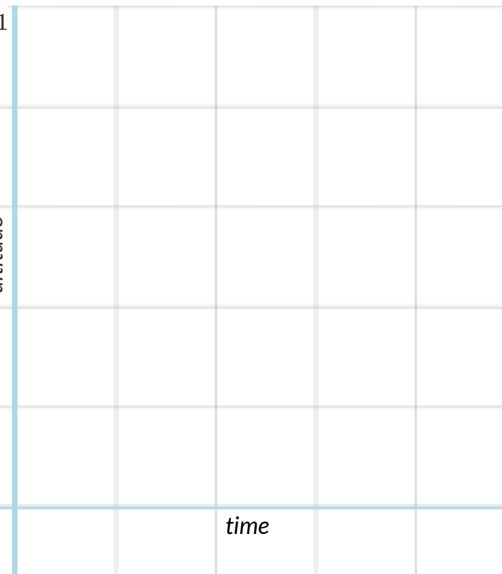
2) Click on one of the ***peaks*** (highest-points) on the graph of your periodic function. Desmos will add a gray dot to *all* of the peaks.

3) Change **ONLY** the slider for b, experimenting with values at 0.2, 0.1, 0.05, and 0, graphing each curve below.
For each curve, label two adjacent peaks.

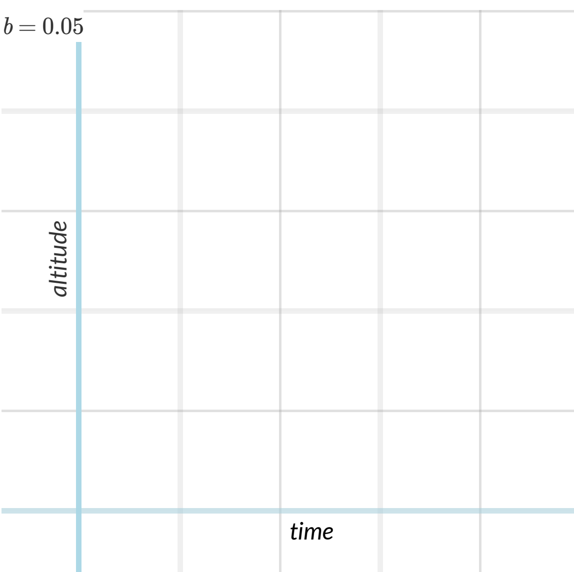
$b = 0.2$



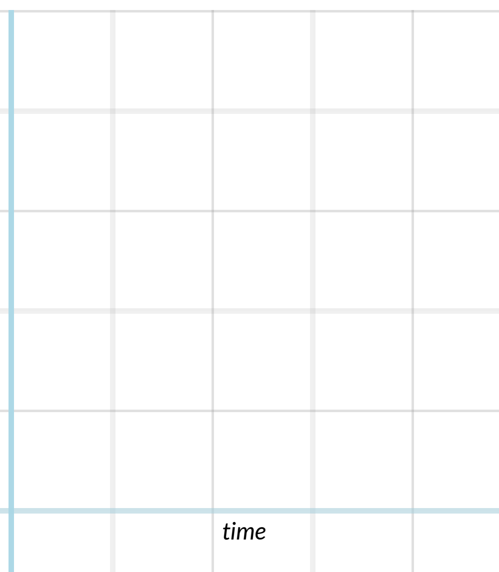
$b = 0.1$



$b = 0.05$



$b = 0$



The distance between two adjacent *peaks* or *troughs* is called the *period*: the interval over which the pattern repeats itself.

4) What is the ***period*** when $b = 0.2$? __________ when $b = 0.1$? __________ When $b = 0.5$? __________ ★ When $b = 0$? ____________

5) As the ***frequency*** (b) doubles, the ***period*** ______________________. As the ***frequency*** (b) gets cut in half, the ***period*** ____________________

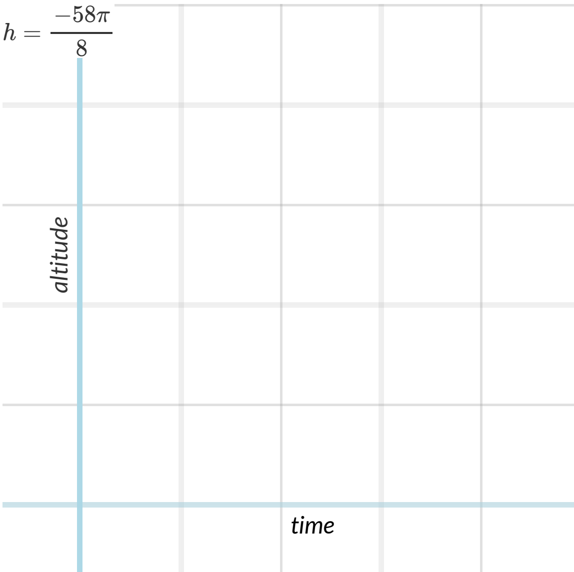
Graphing Periodic Models: Horizontal/Phase Shift (*h*)

The standard form of a periodic model is $f(x) = a\ sin(b{\cdot}(x - h)) + k$. On this page, we'll explore the role of ***amplitude*** a in periodic functions. *Open the Desmos File **Exploring Periodic Functions**. You should be on **Slide 2: Modeling the Ferris Wheel Dataset (sin)** and see four sliders for a, b, h, and k.*

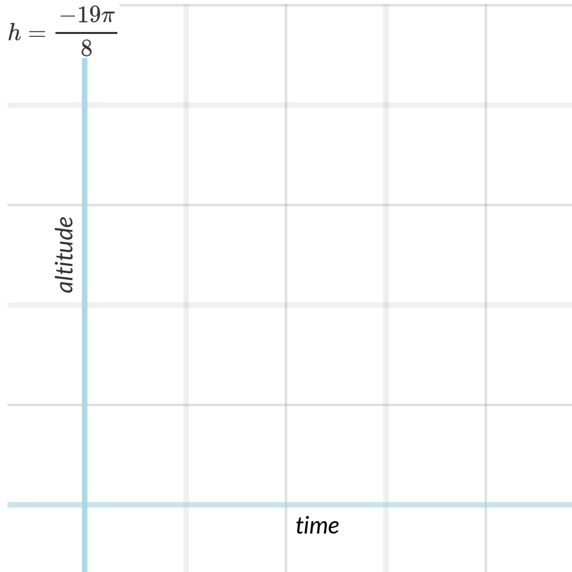
1) Adjust the sliders to fit the data as best you can, and fill in the coefficients: __________ (a), __________ (b), __________ (h) and __________ (k)

2) Change **ONLY** the slider for h, experimenting with values at $\frac{-58\pi}{8}$, $\frac{-19\pi}{8}$, $\frac{19\pi}{8}$, and $\frac{58\pi}{8}$, graphing each curve below. **For each curve, label the coordinates at time=15, 30, and 45.**

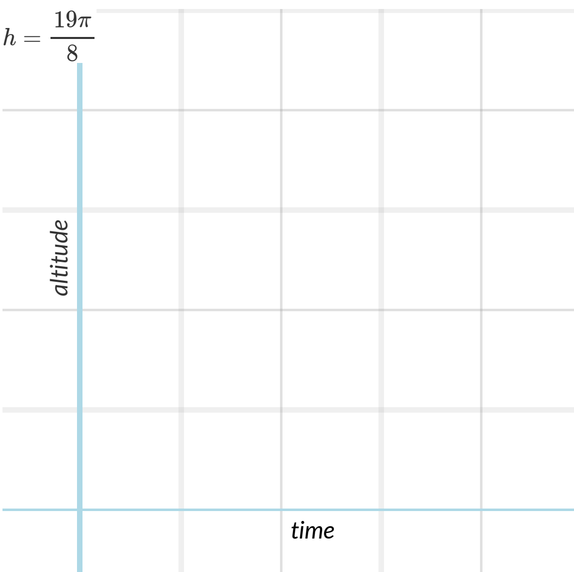
$h = \frac{-58\pi}{8}$



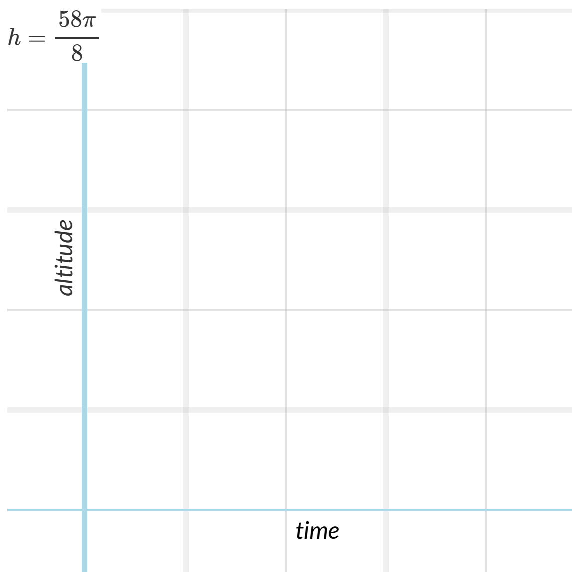
$h = \frac{-19\pi}{8}$



$h = \frac{19\pi}{8}$



$h = \frac{58\pi}{8}$



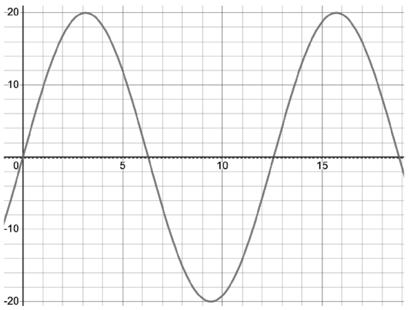
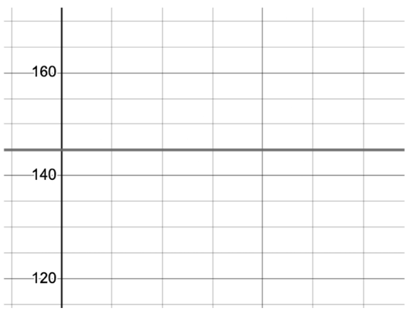
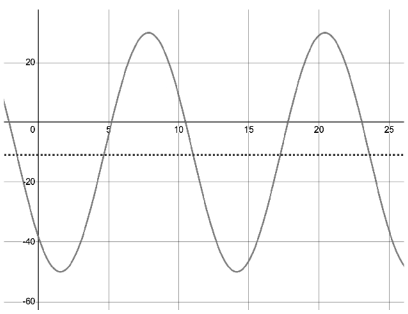
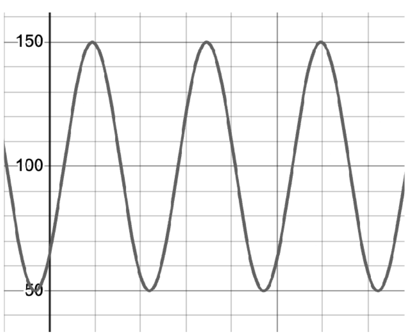
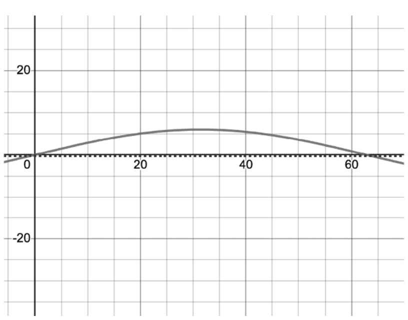
3) Describe the change in the graph when h *increases*: ____________________

4) Describe the change in the graph when h *decreases*: ____________________

5) The model fits as long as h changes by increments of $\frac{77\pi}{8}$, because ____________________

____

Matching Periodic Descriptions

	**1**	**A**	This function has an amplitude of 50
	**2**	**B**	This function has a midline of -10
	**3**	**C**	This function has peaks, troughs, and a midline at 145
	**4**	**D**	This function has a wavelength of more than 60
	**5**	**E**	This function has peaks at 20 and troughs at -20

Modeling the Ferris Wheel Data

Modeling with sin

*For this section, use **Slide 2: "Modeling the Ferris Wheel Dataset (sin)"** of the **Exploring Periodic Functions** Desmos File.* You'll find **the data from the Ferris Wheel plotted in red**, along with a basic periodic model of the form $f(x) = a\sin(b(x - h)) + k$.

1) Use the sliders to estimate the best periodic fit.

2) The ***peaks*** are at ______________ feet, ***troughs*** are at ______________ feet, ***midline*** is at ______________ feet and the ***amplitude*** is ______________ feet

3) The ***period*** of the data is ______________ minutes. If $\text{period} = \frac{2\pi}{\text{frequency}}$, what is the ***frequency***? ______________ cycles per minute

4) Adjust the slider for horizontal shift to find the best fit, then write your model below in Function and Pyret notation. **Express h in terms of pi.**

Function Notation	$f(x) =$ ______ (amplitude) $\times\ sin($ ______ (frequency) (x - ______ (horizontal shift))) + ______ (vertical shift)
Pyret Notation	`fun f(x): (` ______ `* sin(` ______ `* (x -` ______ `))) +` ______ `end`

Translating from sin to cos

*For this section, advance to **Slide 3: "Translating from sin to cos"** of the **Exploring Periodic Functions** Desmos File.* You'll see a function f(x) defined here graphed in blue, which uses cos instead of sin.

5) Adjust the sliders so that the function q perfectly overlaps the function p. What is the value of a? ________ b? ________ k? ________

6) What was the value of h, expressed as a decimal? ______________ What was the value of h, expressed a fraction of pi?

____

7) Change the definition of p in Desmos to math row 1 below and adjust the definition of q to match the new curve. Complete the second row by changing the definition of p in Desmos again and adjust q again.

Function using sin	Function using cos	Vertical Shift k
$p(x) = 10\ sin(1{\cdot}(x - 0)) + 2$	$q(x) =$	
$p(x) =$	$q(x) =$	

8) Do you think that all basic cosine functions can be expressed as sine functions? Why or why not? ______________

____

Modeling with cos

*For this section, advance to **Slide 4: "Modeling the Ferris Wheel Dataset (cos)"** of the **Exploring Periodic Functions** Desmos File.*

9) Translate your sin-based model to a cos-based one. Express the horizontal shift in terms of pi.

Function Notation	$g(x) =$ ______ (amplitude) $\times\ cos($ ______ (frequency) (x - ______ (horizontal shift))) + ______ (vertical shift)
Pyret Notation	`fun g(x): (` ______ `* cos(` ______ `* (x -` ______ `))) +` ______ `end`

Make Your Own Ferris Wheel!

Matching Terms

1) The Ferris Wheel is being upgraded! Match the upgrade on the left to the property that it will change on the right. **NOTE:** some upgrades might change more than one property!

The wheel is being raised *higher*	1	A	***midline***
		B	***vertical shift***
The wheel is being made to spin *faster*	2	C	***frequency***
		D	***amplitude***
The wheel is being made *larger*	3	E	***period***

Design a New Wheel

2) *Design your own Ferris Wheel!* Fill in the table below, then **trade papers with someone else**.

Radius	Altitude of Center	Speed

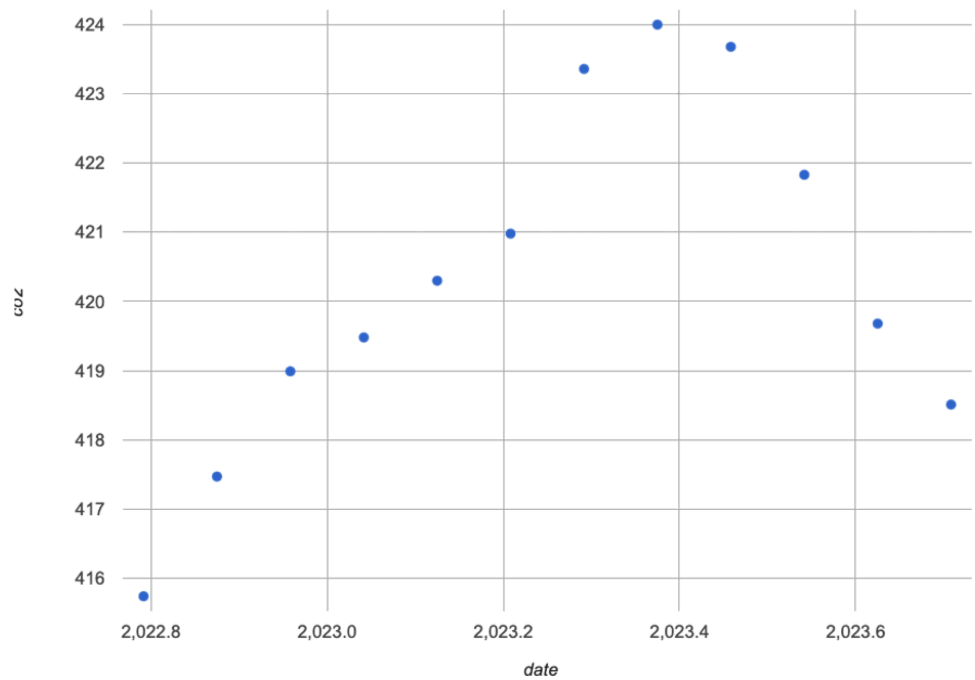
3) Based on the table above, what function will model the height of the wheel over time?

Function Notation	$f(x) = \underset{\text{amplitude}}{\underline{\hspace{8em}}} \times sin(\underset{\text{frequency}}{\underline{\hspace{8em}}}$ (x - $\underset{\text{horizontal shift}}{\underline{\hspace{8em}}}$)) + $\underset{\text{vertical shift}}{\underline{\hspace{8em}}}$
Pyret Notation	`fun f(x): (_____ * sin(_____ * (x - _______))) + _____ end`

Modeling Recent Carbon Dioxide Levels

The data below was generated from the [Carbon Dioxide Starter File](), showing the amount of CO_2 in the atmosphere (parts per million) on specific dates from December 2022 to November 2023. ***NOTE:*** *the* `date` *column is the* ***decimal year*** *(so "June 15th, 2023" would be 2023.5).*

`date`	co2 (ppm)
2022.708	415.91
2022.792	415.74
2022.875	417.47
2022.958	418.99
2023.042	419.48
2023.125	420.30
2023.208	420.98
2023.292	423.36
2023.375	424.00
2023.458	423.68
2023.542	421.83
2023.625	419.68
2023.708	418.51



1) **Connect the dots on the scatter plot to form a line-graph.**

2) The distance between the lowest ***trough*** and highest ***peak*** is ____________ (parts per million), so the ***amplitude*** (a) is ____________ (parts per million)

3) Draw the ***midline*** on your graph. (HINT:look at ***amplitude*** and ***trough***!). What is the ***vertical shift*** (c) of the model? ____________ (parts per million)

4) Estimate the ***phase shift*** by estimating the *decimal year* when the data **first** crosses the ***midline*** (d): ____________ years

5) Calculate the ***period*** between the ***troughs*** by subtracting the `dates` for the lowest values in 2022 and 2023: ____________ (years)

6) If $\text{period} = \dfrac{2\pi}{\text{frequency}}$, what is the ***frequency***? ____________

7) Using your computed values for a, b, h, and k, define your periodic function below in both Function and Pyret notation.

Function Notation	$periodic(x) =$ ________ (amplitude) $\times\ sin($ ________ (frequency) (x - ________ (horizontal shift))) + ________ (vertical shift)
Pyret Notation	`fun periodic(x): (________ * sin(________ * (x - ________))) + ________` `end`

8) Define this model in Pyret, and fit it to the `recent` data. What S-value do you get? ____________

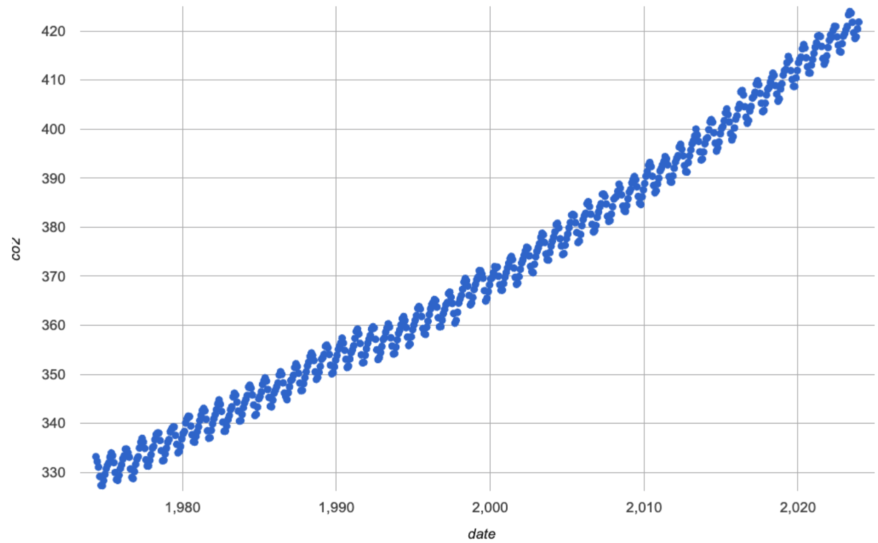
9) **What does this model actually mean?** Fill in the blanks below, and read the completed model aloud with your partner.

Between the end of 2022 and 2023, the amount of CO_2 in the air fluctuated between ________ (lowest) *and* ________ (highest) *parts-per-million. This pattern appears to be* ***periodic****, with an amplitude of* ________ (amplitude) *rising and falling around a* ***midline*** *of* ________ (midline)*. With 1 year representing a full cycle, we expect this pattern to repeat each year for a frequency of* ________ (frequency).

Modeling Historical Carbon Dioxide Levels

*The data below was generated from [Carbon Dioxide Starter File](), showing the amount of CO_2 in the atmosphere (parts per million) on specific dates from 1974 to 2023 (the `co2-table`). **NOTE:** the `date` column is the **decimal year** (so "June 15th, 2023" would be 2023.5).*

Looking for Patterns



1) Use `lr-plot` to find the best linear model for the `co2-table`. What is its S-value? ____________________
parts per million

2) Write the function below (in Pyret and Function Notation):

Function Notation	*linear(x)* = ______ (slope) x + ______ (y-intercept / vertical shift)
Pyret Notation	`fun linear(x): (______ * x) + ______ end`

3) Copy your periodic model from [Modeling Recent Carbon Dioxide Levels]() below:

Function Notation	$periodic(x) =$ ______ (amplitude) $\times$ *sin*(______ (frequency) (x - ______ (horizontal shift))) + ______ (vertical shift)
Pyret Notation	`fun periodic(x): (______ * sin(______ * (x - ______))) + ______ end`

Creating Hybrid Models

We can think of $f(x) = A\sin(B(x - h)) + k$ as being the sum of *two* functions: $p(x) = A\sin(B(x - h))$ and $q(x) = k$.

4) Which function defines the "up and down" wave (p or q)? ________ Which function defines the line the wave "wraps around"? ________

5) What do you think would happen if q were changed so that k is a *higher* number? ____________________

6) What do you think would happen if q were changed so that k is a *lower* number? ____________________

7) What do you think would happen if q were changed to $q(x) = 2x + - 3000$ ____________________

★ Define a NEW function `hybrid` in Pyret, which combines your periodic model with the optimal linear one. Write your new model below, in Function or Pyret notation:

What is the best S-value you can get? ____________________
parts per million

Contracts for Algebra 2

Contracts tell us how to use a function, by telling us three important things:

1. The **Name**
2. The **Domain** of the function - what kinds of inputs do we need to give the function, and how many?
3. The **Range** of the function - what kind of output will the function give us back?

For example: The contract `triangle :: (Number, String, String) -> Image` tells us that the name of the function is `triangle`, it needs three inputs (a Number and two Strings), and it produces an Image.

With these three pieces of information, we know that typing `triangle(20, "solid", "green")` will evaluate to an Image.

Name		Domain		Range
# above	::	(Image [above], Image [below])	->	Image
above(circle(10, "solid", "black"), square(50, "solid", "red"))				
# bar-chart	::	(Table [table-name], String [column])	->	Image
bar-chart(animals-table, "species")				
# box-plot	::	(Table [table-name], String [column])	->	Image
box-plot(animals-table, "weeks")				
# build-column	::	(Table [table-name], String [column], (Row -> Value) [builder-function])	->	Table
build-column(animals-table, "kilos", kilograms)				
# count	::	(Table [table-name], String [column])	->	Table
count(animals-table, "species")				
# first-n-rows	::	(Table [table-name], Number [num-rows])	->	Table
first-n-rows(animals-table, 15)				
# fit-model	::	(Table [table-name], String [labels], String [xs], String [ys], (Num -> Num) [model-function])	->	Image
fit-model(animals-table, "name", "pounds","weeks", f)				
# histogram	::	(Table [table-name], String [labels], String [values], Number [bin-size])	->	Image
histogram(animals-table, "species", "weeks", 2)				
# line-graph	::	(Table [table-name], String [labels], String [xs], String [ys])	->	Image
line-graph(animals-table, "name", "pounds","weeks")				
# log	::	(Number [n])	->	Number
log(4)				
# log-base	::	(Number [base], Number [n])	->	Number
log-base(2, 4)				

Name		Domain		Range
# lr-plot	::	(Table (table-name), String (labels), String (xs), String (ys))	->	Image
`lr-plot(animals-table, "name", "pounds","weeks")`				
# num-sqr	::	(Number)	->	Number
`num-sqr(4)`				
# overlay	::	(Image (top), Image (bottom))	->	Image
`overlay(circle(10, "solid", "black"), square(50, "solid", "red"))`				
# pie-chart	::	(Table (table-name), String (column))	->	Image
`pie-chart(animals-table, "species")`				
# put-image	::	(Image (front), Number (x-coordinate), Number (y-coordinate), Image (behind))	->	Image
`put-image(circle(10, "solid", "black"), 10, 10, square(50, "solid", "red"))`				
# rotate	::	(Number (degrees), Image (img))	->	Image
`rotate(45, star(50, "solid", "dark-blue"))`				
# row-n	::	(Table (table-name), Number (index))	->	Row
`row-n(animals-table, 2)`				
# S	::	(Table (table-name), String (xs), String (ys), (Num -> Num) (model-function))	->	Number
`S(animals-table, "name", "pounds","weeks", f)`				
# scale	::	(Number (factor), Image (img))	->	Image
`scale(1/2, star(50, "solid", "light-blue"))`				
# scatter-plot	::	(Table (table-name), String (labels), String (xs), String (ys))	->	Image
`scatter-plot(animals-table, "name", "pounds","weeks")`				
# sort	::	(Table (table-name), String (column), Boolean (ascending))	->	Table
`sort(animals-table, "species", true)`				
# string-contains	::	(String (haystack), String (needle))	->	Boolean
`string-contains("hotdog", "dog")`				
	::		->	
	::		->	
	::		->	

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